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Delay and Capacity Trade-offs in Mobile Wireless Networks with Infrastructure Support

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Abstract In this paper, we investigate the trade-offs between delay and capacity in mobile wireless networks with infrastructure support. We consider three different mobility models, independent and identically distributed (i.i.d) mobility model, random walk mobility model with constant speed and Lévy flight mobility model. For i.i.d mobility model and random walk mobility model with the speed $\Theta(\frac{1}{\sqrt{n}})$, we get the theoretical results of the average packet delay when capacity is $\Theta(1)$, $\Theta(\frac{1}{\sqrt{n}})$ individually, where n is the number of nodes. We find that the optimal average packet delay is achieved when capacity $\lambda(n) < \frac{1}{2\cdot n\cdot\log_2(\frac{1}{1-e^{-\frac{K}{n}}}+1)}$, where K is the number of gateways. It is proved that average packet delay $D(n)$ divided by capacity $\lambda(n)$ is bounded below by $\frac{n}{K\cdot W}$. When $\omega(\sqrt{n}) \leqslant K < n$, the critical average delay for capacity compared with static hybrid wireless networks is $\Theta(\frac{K^2}{n})$. Lévy flight mobility model is based on human mobility and is more sophisticated. For the model with parameter α, it is found that $\frac{D(n)}{\lambda(n)} > O(n^{\frac{(1-\eta)\cdot(\alpha+1)}{2}}\ln n)$ when $K = O(n^\eta)$ $(0 \leqslant \eta < 1)$. We also prove that when $\omega(\sqrt{n}) \leqslant K < n$, the critical average delay is $\Theta(n^{\frac{\alpha-1}{2}} \cdot K)$.

Keywords mobile wireless networks, capacity, delay

1 Introduction

With the development of wireless technologies, different wireless access methods have been integrated on a single device. Users can connect to the Internet through one-hop wireless LANs (WLANs), or multi-hop wireless networks. Recently, as an effective complement to the static infrastructure wireless networks, mobile wireless networks based on opportunistic transmission have been investigated[1-3].

The mobile users can connect to the Internet through gateways, which are usually formed as infrastructure networks. When a user moves into the transmission range of a gateway, useful data is downloaded. Users can share data in "store-carry-and-forward" manner. When two nodes move close enough, interested data can be exchanged. Because such an opportunistic transmission is usually done through WiFi or Bluetooth, it is free of charge and generates no traffic load on the infrastructure network. Through analysis on real trace data and simulation results, it is found that such mobile wireless networks with infrastructure support improve performance and energy efficiency significantly[4-5].

To evaluate this kind of mobile wireless networks with infrastructure support theoretically, we consider two performance metrics, capacity of the network and average packet delay. The former represents the expected number of packets that can be delivered successfully in unit time, and the latter represents the average time it takes for the transmission of a packet. Huang *et al.* investigated capacity of mobile wireless networks with infrastructure support[6]. They adopted a mobility model where nodes move around

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their home-points. Different with the one-hop wireless communication they considered, we focus on multi-hop opportunistic transmission. Besides capacity, we investigate another important metric delay.

Relationship between these two metrics will provide useful insights on the characteristics of network performance. Existing work has shown that there is a trade-off between them in mobile wireless networks without infrastructure support[7]. The introduction of infrastructure makes the problem more difficult because there are two different transmission modes for all pairs of source and destination. One is without the help of gateways, which is similar to the opportunistic transmission. The other is through gateways, which is like the traditional infrastructure based transmission.

In this paper, we investigate delay-capacity trade-offs in such mobile hybrid wireless networks under independent and identically distributed (i.i.d) mobility model, random walk mobility model with constant speed and Lévy flight mobility models individually. The first two are the most used synthetic mobility models, and the last one is proposed more recently based on the human mobility characteristics[8].

Our main contributions are as follows.

- We give the results of the average packet delay under i.i.d mobility model when capacity is $\Theta(1)$①, $\Theta(\frac{1}{\sqrt{n}})$ individually. It is proved that average packet delay $D(n)$ can be optimized when the capacity $\lambda(n) < \frac{1}{2\cdot n\cdot \log_2(\frac{1}{1-e^{-\frac{K}{n}}}+1)}$, where n is the number of nodes and K is the number of gateways.
- It is found that in this kind of network under i.i.d mobility model, $\frac{D(n)}{\lambda(n)} > \frac{n}{K\cdot W}$. Compared with the static hybrid wireless networks, it is found that the critical average delay is $\Theta(\frac{K^2}{n})$ when $\omega(\sqrt{n}) \leqslant K < n$.
- It is proved that, under random walk mobility model with speed $\Theta(\frac{1}{\sqrt{n}})$, delay and capacity can achieve the same trade-offs in scale sense with those under i.i.d mobility model.
- Under Lévy flight mobility model with parameter α, we prove that $\frac{D(n)}{\lambda(n)} > O(n^{\frac{(1-\eta)\cdot(\alpha+1)}{2}}\ln n)$ if $K = O(n^{\eta})$ $(0 \leqslant \eta < 1)$. When $\omega(\sqrt{n}) \leqslant K < n$, the critical average delay is $\Theta(n^{\frac{\alpha-1}{2}}\cdot K)$.

The rest of the paper is organized as follows. Section 2 reviews the related work. Section 3 presents the models we use and the problem we investigate. We give our main results in Section 4. Finally, concluding remarks are given in Section 5.

2 Related Work

In the landmark paper, Gupta and Kumar showed that throughput per-node scales as $\Theta(\frac{1}{\sqrt{n\log n}})$ in random static wireless networks[9]. It can be increased to $\Theta(1)$ when mobility is introduced[10]. However, [10] does not address the delay issue.

Since then, there has been substantial work investigating the trade-offs between delay and capacity in mobile wireless networks under different mobility models. In [11], mobility characteristics is exploited and a routing algorithm is designed which approaches the optimal capacity while keeping the delay small. In [12-15], the problem is studied under i.i.d mobility model. Neely *et al.* proposed a packet scheduling scheme to achieve the trade-off $\lambda(n) \leqslant O(\frac{D(n)}{n})$, where $\lambda(n)$ is the throughput per-node, $D(n)$ is the average end-to-end delay and n is the number of nodes in the network[12]. Incorporating the multiuser reception and power control, a better capacity-delay trade-off $\lambda^2(n) \leqslant \Theta(\frac{D(n)}{n})$ is achieved[13]. In [14], the upper bound of the maximum throughput per-node under a given delay constraint is established, and the scheduling schemes which can approach the bound up to some logarithmic factor are proposed. Liu *et al.* gave the closed-form theoretical results for two-hop relaying based mobile wireless networks[15]. In [16-17], the problem is investigated under Brownian motion model. A scheme to achieve the optimal order of delay for any given throughput is described in [16]. [18-20] discuss the problem under random walk mobility model and restricted mobility model individually. From a global perspective, Sharma *et al.* established the relationship between critical delay and first exit/hitting time[7]. Critical delay means the minimum delay that has to be tolerated in mobile networks to achieve the same throughput in scale sense in static wireless networks.

Different new mobility models have appeared and the corresponding delay-capacity trade-offs have been investigated. Ying *et al.* proposed four new mobility models, and investigated the maximum throughput per-node with a delay constraint[21]. Garetto *et al.* analyzed the results under the mobility models where each node moves around its home-points[22-23]. In [24], it is found that spatial heterogeneity improves the delay-capacity scaling laws. Assuming home-points can be wired-connected, Huang *et al.* gave the scaling laws of capacity[6]. In [26], the trade-off under the reference point group mobility model[25] is investigated, and it is shown that the movement relevance of different nodes improves the performance. More recently, Wang *et al.* gave the asymptotic capacity and delay bounds for two different mobility models under Gaussian Channel Model[27]. The two mobility models are hybrid random walk mobility model and discrete random direction

①We use standard order notations.

mobility model. Based on the work of [7], Lee *et al.* derived the critical delay under Lévy mobility model[28].

Above work mainly focuses on unicast, Li derived the asymptotic bounds of multicast capacity in wireless networks under protocol interference model[29]. The results generalize the ones of unicast[9] and broadcast[30-31]. Li and Liu *et al.* also gave the bounds under Gaussian Channel Model[32]. Wang *et al.*[33] found that the ratio between delay and capacity in multicast is smaller than that directly extending the result in [12]. Targeting at the mobility models similar to [21], they gave a global perspective of the delay-capacity trade-off of multicast in mobile wireless networks[34].

Different with the existing work, we take both mobility and infrastructure support into consideration, which are the two important aspects that will impact network performance. Besides the two traditional mobility models which are used most in simulations and analysis, we also discuss the Lévy flight mobility model which is important for the investigation of human mobility.

3 Models and Assumptions

The network we investigate consists of n nodes and K $(1 \leqslant K < n)$ gateways in a square of unit area. Initially, they are all uniformly distributed. Time is divided into slots and all nodes move following the same mobility model in each slot.

3.1 Mobility Models

We investigate three different models in this paper.

• *i.i.d Mobility Model.* In the beginning of each slot, nodes uniformly select a random position in the square and stay there for the rest of the slot.

• *Random Walk Mobility Model with Constant Speed.* In the beginning of each slot, nodes uniformly select a random direction in $[0, 2\pi]$ and keeps moving with the same speed S for the rest of the slot.

• *Lévy Flight Mobility Model.* Different with the above models, in the beginning of each odd slot, every node uniformly selects a random direction in $[0, 2\pi]$ and moves with a chosen speed. In even slots, nodes stay at the position where it arrived by the end of the previous slot. Speed S_i of node i follows Lévy distribution. Its probability density function (PDF) is $p(s) = \frac{1}{2\pi}\int_{\frac{1}{\sqrt{n}}}^{1} e^{-its-|Ct|^{\alpha}}\mathrm{d}t$, where C is a scale factor and $\alpha \in (0, 2]$.

The first two models are traditional[35]. The third one is used to model human mobility. Studies on real mobility data show that flight length of human mobility has a power-law tail[8,36], where flight means the longest straight line trip from one position to the other in each slot. Because the network we consider is in a unit square, we assume the flight length is bounded below by $\frac{1}{\sqrt{n}}$ and above by 1.

We assume the above three models belong to fast mobility[34], and in each time slot, only one-hop transmission is allowed.

3.2 Transmission Model

Each node i in the network can perform as a source, and the corresponding destination node is denoted as $d(i)$. Sources generate traffic with constant rate continuously. They can deliver data to the destinations directly or with the help of relays. We make the same assumption as [10] that all nodes can perform as relays and they have infinite buffer space.

We assume all nodes use the same channel or code. When two nodes are in the transmission range $r(n)$ of each other and there is no interfering traffic, they can communicate successfully with W *bits per second* (bps) on a common channel. We adopt the protocol interference model[9]. Transmission from nodes i to j is interfered by that from node k iff $d_{k,j} \leqslant (1 + \Delta) \cdot d_{i,j}$, where $d_{k,j}$ is the distance between the two nodes k, j, $d_{i,j}$ is that between i, j, and Δ is a parameter predefined. Interfering transmission on different links can be scheduled by an MAC protocol following time division multiple access (TDMA).

There are usually two ways for a destination to get data: *neighbor-capture* and *multi-hop capture*[7,28]. In the former, it gets data when the source or a relay node carrying the data is in its neighborhood. In the latter, when there is a multi-hop path between it and the source, data is successfully delivered and transmission delay is negligent compared with the time interval between two consecutive appearance of a path. Because we assume the mobility models belong to fast mobility, neighbor-capture is adopted in this paper.

3.3 Performance Metrics

We investigate two performance metrics *capacity of the network* and *average packet delay*, which are defined as follows.

Definition 1 (Capacity of the Network). *Capacity* $\lambda(n)$ *is*

$$\lambda(n) = \lim_{t\to\infty} \inf \frac{1}{n}\sum_{i=1}^{n}\frac{\lambda_i(t)}{t}, \tag{1}$$

where $\lambda_i(t)$ *is the number of packets delivered successfully from node* i *to* $d(i)$ *in the time period of length* t.

Definition 2 (Average Packet Delay). *Delay $D(n)$ is*

$$D(n) = \lim_{m\to\infty} \frac{1}{n}\sum_{i=1}^{n}\frac{1}{m}\sum_{j=1}^{m} D_{i,j}, \tag{2}$$

where $D_{i,j}$ is the time it takes for the j-th packet from node i to be delivered successfully to $d(i)$.

In this paper, we investigate the fundamental relationship between delay and capacity under different mobility models in the mobile hybrid wireless networks.

4 Main Results

4.1 i.i.d Mobility

First we investigate how the average packet delay scales when the capacity is $\Theta(1)$ under i.i.d mobility model. Fig.1 shows the movement of four nodes.

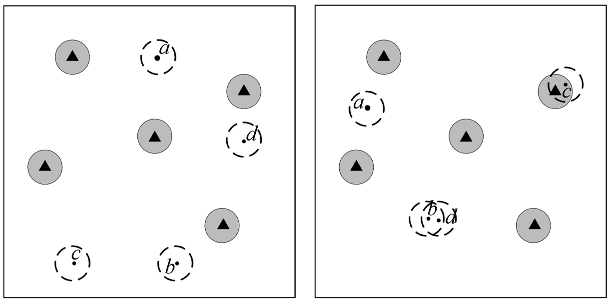

Fig.1. Four nodes (a, b, c, and d) following i.i.d mobility model move in two time slots. The gateways (denoted as the triangles) are uniformly distributed in the square.

Because the transmission range of nodes can be adjusted according to different requirements (when power control is adopted), for the convenience of analysis, firstly we assume that $r(n) = \frac{1}{\sqrt{\pi n}}$. For every source-destination (S-D) pair, if the source only transmits packets directly to destination, due to the long delay for them to meet, capacity will not be optimized. We consider a simple two-hop routing strategy in which only one copy of each packet is kept, and denote it as Strategy 2H-I (two hop one copy). In this strategy, each time slot is divided into two sub-slots evenly. We divide the square into $\frac{\pi n}{(1+\Delta)^2}$ cells (for simplicity, it is assumed that $\frac{\pi n}{(1+\Delta)^2}$ is an integer). In odd sub-slots a node in each cell is randomly selected to perform as source, and the other nodes and gateways in the same cell perform as relays. In even sub-slots, considering a relay node, if in the same cell there are destinations for the packets it is carrying in its transmission range, it randomly selects one and delivers the data in first come first served (FCFS) order. If there is a gateway in its transmission range, the node will treat all the packets it is carrying as in the same queue, and try to deliver them to the gateway in FCFS order.

Lemma 1. *In Strategy 2H-I, $\lambda(n) = \Theta(1)$.*

Proof. For a pair of nodes in communication, it is assumed that the nodes in the disk with radius $\frac{1+\Delta}{\sqrt{\pi n}}$ centered at the receiver (sender) do not send (receive) data at the same time in order to avoid interference. The expected number of cells where there are successful transmissions simultaneously is bounded below by $\frac{\pi n}{9(1+\Delta)^2}$. In each cell, the expected number of nodes is $\Theta(1)$. Thus, the expected number of nodes that can transmit data simultaneously is $\Theta(n)$. In every odd time slot, all sources can deliver $\Theta(1)$ packets.

For a special S-D pair, the probability that a relay node meets the destination in a slot is $\frac{1}{n}$. There are $\Theta(n)$ nodes which can perform as relays. When time is long enough, all destinations can receive $\Theta(\frac{1}{n}\cdot n) = \Theta(1)$ packets for a particular S-D pair in every even slot. So, capacity $\Theta(1)$ is achievable. □

Given a strategy by which $\Theta(1)$ capacity is achieved, we discuss the resulted average packet delay.

We denote packets transmitted with the help of gateways as P_g and their delay as D_g. The others are P_s and the corresponding delay is D_s. D_g and D_s both include two parts: time needed to establish a path between the source and destination, and the waiting time in queues. We have Lemma 2.

Lemma 2. *In Strategy 2H-I, the expected delay $E(D_g) = O(\frac{1}{1-e^{-\frac{K}{n}}})$, $E(D_s) = O(n)$.*

Proof. We use T_s to represent the time it takes to establish a path between the source and destination for packets in P_s. T_g represents that for packets in P_g. For the convenience of analysis, we assume that every packet is firstly sent to a mobile relay node from the source, neither the destination nor the gateways. In this case, T_s constitutes of two part: the time from the source to relay and that from the relay to the destination. T_g constitutes of three parts: the time from the source to a relay, that from the relay to a gateway, and that from gateways to the destination. It takes only half a slot for the packet to be delivered from the source to a relay, and in the following sub-slot the relay will try to deliver it to the destination. T_g can be regarded as following the negative binomial distribution with parameters $(2, 1-e^{-\frac{K}{n}})$, and T_s follows geometric distribution with the success probability $\frac{1}{n}$. Their expected value and variance are

$$E(T_s) = n, \quad D(T_s) = \frac{1-\dfrac{1}{n}}{\dfrac{1}{n^2}} = n^2 - n,$$

$$E(T_g) = \frac{2(1-e^{-\frac{K}{n}})}{e^{-\frac{K}{n}}}, \quad D(T_g) = \frac{2(1-e^{-\frac{K}{n}})}{e^{-\frac{2K}{n}}}.$$

Every node maintains a separate queue for each S-D pair, and packet transmissions with and without gateways can be treated as two different kinds of service. The queues are GI/GI/1-FCFS. The average waiting time $E(W^q)$ for a GI/GI/1-FCFS queue satisfies

$$E(W^q) \leqslant \frac{\Delta^1 + \Delta^2}{\mu^1 - \mu^2}, \tag{3}$$

where μ^1 and μ^2 are the stationary independent inter-arrival and inter-departure time, Δ^1 and Δ^2 are the variances.

For packets belonging to P_s, μ_s^1 equals the expected meeting time between the source node and relay node. In the i.i.d mobility, the meeting time between two nodes is geometrically distributed with success probability $\frac{1}{n}$. The source can control the transmission rate to guarantee $\mu_s^1 = \epsilon \cdot n(\epsilon > 1)$ and $\Delta_s^1 = \epsilon^2 \cdot n^2 - \epsilon^3 \cdot n$.

A mobile node can relay packets for every S-D pair. When it meets the gateways, it treats all the packets to relay as in a single queue and serves them in the FCFS order. Because the bandwidth between the node and a gateway is W, the proportion of packets belonging to P_g for each S-D pair is bounded above by $\frac{K \cdot W}{n \cdot \lambda(n)} = \Theta(\frac{K}{n})$, and is denoted as p_g. When a source delivers a packet to a relay node, we assume the probability that will be transmitted through gateways is p_g. The first meeting time between a relay node and any other node is geometrically distributed with success probability $1-(1-\frac{1}{n})^{n-1}$. For the inter-arrival time of packets belonging to P_g, when n approaches to infinity we have

$$\mu_g^1 = \frac{1}{1-\left(1-\dfrac{p_g}{n}\right)^{n-1}} = \frac{1}{1-e^{\frac{-p_g\cdot(n-1)}{n}}} = \frac{1}{1-e^{-p_g}}, \tag{4}$$

$$\Delta_g^1 = \frac{\left(1-\dfrac{p_g}{n}\right)^{n-1}}{\left(1-\left(1-\dfrac{p_g}{n}\right)^{n-1}\right)^2} = \frac{e^{-p_g}}{(1-e^{-p_g})^2}. \tag{5}$$

The expected inter-departure time and the corresponding variances are

$$\mu_s^2 = E(T_s), \quad \Delta_s^2 = D(T_s),$$
$$\mu_g^2 = E(T_g), \quad \Delta_g^2 = D(T_g).$$

Thus, there are two different average waiting time for the packets,

$$E(W_q^s) \leqslant O(n),$$
$$E(W_q^g) \leqslant O\Big(\frac{1}{1-e^{-\frac{K}{n}}}\Big).$$

The two different average delays for the packets are

$$E(D_s) = E(W_q^s) + \mu_2^s = O(n),$$

$$E(D_g) = E(W_q^g) + \mu_2^g = O\Big(\frac{1}{1-e^{-\frac{K}{n}}}\Big). \qquad \square$$

Based on Lemmas 1 and 2, we have

Theorem 1. *In i.i.d mobility model, if $K = o(n)$, $\lambda(n) = \Theta(1)$ is achieved when $D(n) = O(n)$; if $K = \Theta(n)$, $\lambda(n) = \Theta(1)$ is achieved when $D(n) = O(1)$.*

Proof. The proportion of packets belonging to P_g for each S-D pair is bounded above by $p_g = \Theta(\frac{K}{n})$. When $K = o(n)$, the average delay for the packets in Strategy 2H-I is

$$D(n) = p_g \cdot E(D^g) + (1-p_g) \cdot E(D^s) = O(n).$$

According to Lemma 1, we have that in this situation $\lambda(n) = \Theta(1)$ is achieved when $D(n) = O(n)$.

When $K = \Theta(n)$, we use a new strategy which is different with Strategy 2H-I in that none of the mobile nodes relay packets. Only the gateways perform as relays. It can be easily proved that the source and destination in an S-D pair will meet a gateway in $\Theta(1)$ slots. The capacity $\lambda(n) = \Theta(1)$ can be achieved. According to Lemma 2, $D(n) = O(\frac{1}{1-e^{-\frac{K}{n}}}) = O(1)$ in this strategy. $\square$

Average packet delay may be improved at the cost of decrease of capacity. For every packet, if they are replicated more, the probability that the destination gets a copy successfully will be higher. We consider a new transmission strategy which is different with Strategy 2H-I. The source keeps transmitting copies of a packet, until it meets the destination node or the number of copies reaches to $\sqrt{n}$. The $\sqrt{n}$ mobile relay nodes deliver the copies to gateways or destination they meet. They do not deliver copies to other mobile nodes. Relays keep a queue for each S-D pair. A timestamp is attached on every packet denoting the time when it is generated at the source. When a relay meets a destination, it first schedules the packets it is carrying which is generated earliest. When it meets a gateway, all packets in the queues for different S-D pairs will be treated as in the same queue, and the one with the earliest timestamp will be delivered first. We denote this strategy as Strategy 2H-M (two hops multiple copies).

Lemma 3. *In Strategy 2H-M, considering the packet with the earliest timestamp of an S-D pair, its expected transmission time $E(D_s)$ only through mobile relay nodes satisfies $E(D_s) \leqslant O(\sqrt{n})$. When $\omega(\sqrt{n}) \leqslant K < n$, its expected transmission time $E(D_g)$ with the help of gateways satisfies $E(D_g) \leqslant O(\frac{1}{1-e^{-\frac{K}{n}}})$.*

Proof.

$$E(D_s) \leqslant E \text{ (time needed for relays to get copies)} +$$
$$E \text{ (delay from } \sqrt{n} \text{ relays to destination)}$$

$$\leqslant \sqrt{n} + \frac{1}{1-\left(1-\frac{1}{n}\right)^{\sqrt{n}}} \leqslant \sqrt{n} + \frac{1}{1-e^{-\frac{1}{\sqrt{n}}}} = O(\sqrt{n}).$$

Considering the upper bound for $E(D_g)$, we denote the expected transmission time from source to gateways as $E(T_{s\to g})$, that from gateways to destination as $E(T_{g\to u})$, and $T_g = T_{s\to g} + T_{g\to u}$. For convenience of calculation, we assume that the gateways only receive copies from mobile relays, and not from the source directly. Under this assumption, the delay we get is an upper bound, which we are concerned about.

$$\begin{aligned}
&\Pr\left(T_{s\to g} \leqslant t\right)\\
&= \begin{cases} 1-\left(1-\frac{1}{n}\right)^{K\cdot t+K\cdot(t-1)+\cdots+K}, & \text{if } t \leqslant \sqrt{n},\\ 1-\left(1-\frac{1}{n}\right)^{K\cdot t+K\cdot(t-1)+\cdots+K\cdot(t-\sqrt{n})}, & \text{otherwise}\end{cases}\\
&= \begin{cases} 1-\left(1-\frac{1}{n}\right)^{K\cdot\frac{t\cdot(t+1)}{2}}, & \text{if } t \leqslant \sqrt{n},\\ 1-\left(1-\frac{1}{n}\right)^{K\cdot\frac{(2t-\sqrt{n})\cdot(\sqrt{n}+1)}{2}}, & \text{otherwise}\end{cases}\\
&> \begin{cases} 1-\left(1-\frac{1}{n}\right)^{K\cdot\frac{t^2}{2}}, & \text{if } t \leqslant \sqrt{n},\\ 1-\left(1-\frac{1}{n}\right)^{K\cdot\frac{2t\cdot\sqrt{n}-n}{2}}, & \text{otherwise}\end{cases}\\
&> \begin{cases} 1-e^{-\frac{K\cdot t^2}{2n}}, & \text{if } t \leqslant \sqrt{n},\\ 1-e^{-\frac{K\cdot\frac{2t\cdot\sqrt{n}-n}{2}}{n}}, & \text{otherwise.}\end{cases}
\end{aligned}$$

If $K = \omega(1)$, when $t = K^{\frac{\beta-1}{2}}\cdot\sqrt{2n}$, $\beta > 0$,

$$\begin{aligned}
&\lim_{n\to\infty} \Pr\left(T_{s\to g} \leqslant t\right) = 1,\\
&E(T_{s\to g}) \leqslant K^{\frac{\beta-1}{2}}\cdot\sqrt{2n}.\\
&E(T_{g\to u}) \leqslant \frac{1}{1-(1-\frac{1}{n})^K} \leqslant \frac{1}{1-e^{-\frac{K}{n}}},\\
&E(T_g) \leqslant K^{\frac{\beta-1}{2}}\cdot\sqrt{2n} + \frac{1}{1-e^{-\frac{K}{n}}}.
\end{aligned}$$

It is found that the upper bound of $E(T_g)$ is dominated by that of $E(T_{g\to u})$ when $K = \omega(1)$. We have

$$E(T_g) \leqslant O\Big(\frac{1}{1-e^{-\frac{K}{n}}}\Big), \tag{6}$$

$$D(T_g) \leqslant O\Big(\frac{e^{-\frac{K}{n}}}{(1-e^{-\frac{K}{n}})^2}\Big). \tag{7}$$

A relay node may carry packet copies of different S-D pairs. When it meets a gateway, it delivers them in the order according to their generation time. For a mobile relay node, we can regard it and the K gateways as a server. The server serves for all the $n-1$ nodes. Packets whose transmission through them will experience a period of waiting time in the queue. (6) and (9) can be used as the upper bound of the expected value and variance of inter-departure time. Similar to (4) and (5), when n approaches infinity the expected value and variance of inter-arrival time are

$$\mu_g^1 = \frac{1}{1-\left(1-\frac{p_g}{\sqrt{n}\cdot n}\right)^{n-1}} = \frac{1}{1-e^{-\frac{p_g}{\sqrt{n}}}}, \tag{8}$$

$$\Delta_g^1 = \frac{\left(1-\frac{p_g}{\sqrt{n}\cdot n}\right)^{n-1}}{\left(1-\left(1-\frac{p_g}{\sqrt{n}\cdot n}\right)^{n-1}\right)^2} = \frac{e^{-\frac{p_g}{\sqrt{n}}}}{(1-e^{-\frac{p_g}{\sqrt{n}}})^2}, \tag{9}$$

where p_g is bounded above by $\min(\frac{K\cdot W}{n\cdot\lambda(n)}, 1)$. Because in this strategy, sources send $\sqrt{n}$ copies for each packet, $p_g = \min(\frac{K\cdot W}{\sqrt{n}}, 1)$. When $\omega(1) \leqslant K \leqslant O(\sqrt{n})$, the upper bound of $E(T_g)$ is bounded below by $O(\sqrt{n})$. The introduction of gateways does not improve the upper bound in scale sense. Thus, we consider $\omega(\sqrt{n}) \leqslant K < n$. In this case, $p_g = 1$ and

$$\begin{aligned}
&\lim_{n\to\infty} \frac{\mu_g^1}{E(T_g)} = \infty,\\
&E(D_g) = E(T_g) \leqslant O\Big(\frac{1}{1-e^{-\frac{K}{n}}}\Big).
\end{aligned} \tag{10}$$

□

Getting the average packet delay in Strategy 2H-M, we investigate the resulted capacity. We have

Theorem 2. *In i.i.d mobility model, capacity $\lambda(n) = O(\frac{1}{\sqrt{n}})$ can be achieved when the average packet delay $D(n) = O(\sqrt{n})$ if $K = O(\sqrt{n})$. If $\omega(\sqrt{n}) \leqslant K < n$, capacity $\lambda(n) = O(\frac{K}{n})$ can be achieved when $D(n) = O(\frac{1}{1-e^{-\frac{K}{n}}})$.*

Proof. In Strategy 2H-M, the expected transmission time T for a single packet satisfies

$$\begin{aligned}
E(T) &\leqslant \min\Big(O(\sqrt{n}), O\Big(\frac{1}{1-e^{-\frac{K}{n}}}\Big)\Big)\\
&= \begin{cases} O(\sqrt{n}), & K = O(\sqrt{n}),\\ O\Big(\frac{1}{1-e^{-\frac{K}{n}}}\Big), & \omega(\sqrt{n}) \leqslant K < n.\end{cases}
\end{aligned}$$

When $K = O(\sqrt{n})$, we just consider the packet transmission without the help of gateways. Because relays just hold copies and do not send them to other mobile nodes (except the destination), the transmission of an S-D pair has no influence on that of the other pairs. Transmission of different pairs can be regarded as receiving service from different servers. According to Lemma 2 in [12], $D(n) = E(T)$ when $\frac{1}{O(\sqrt{n})} \leqslant \lambda(n) <$

$\frac{1}{E(T)}$. When $\omega(\sqrt{n}) \leqslant K < n$, $p_g = \Theta(1)$ and we just consider the packet transmission through gateways. According to (10), the waiting time to access the gateways for packets of different S-D pairs on a relay node can be omitted. The service of different S-D pairs can also be regarded irrelevant, and we have that $\lambda(n)$ can be bounded below by $1 - e^{-\frac{K}{n}}$. However, because the source keeps sending $\sqrt{n}$ copies for each packet, $\lambda(n)$ is bounded above by $O(\frac{1}{\sqrt{n}})$.

It can be easily found that when $\omega(\sqrt{n}) \leqslant K < n$, there is a gap between $1-e^{-\frac{K}{n}}$ and $\frac{1}{\sqrt{n}}$. We change the number of copies the source keeps sending for a packet in Strategy 2H-M, from $\sqrt{n}$ to $\frac{n}{K}$. Following similar proof of Lemma 3, we get the new expected transmission time for a packet is still $\frac{1}{1-e^{-\frac{K}{n}}}$. The capacity $\lambda(n) = O(\max(1 - e^{-\frac{K}{n}}, \frac{K}{n})) = O(\frac{K}{n})$. □

The more copies of a packet there are in the mobile network, the shorter its transmission delay is. For a single packet, optimal transmission delay is achieved when epidemic routing is used, in which every node will send copies of the packet to others nodes in every transmission opportunity if possible.

We propose an epidemic routing based strategy and denote it as Strategy MH-M (multiple hops multiple copies). The difference with Strategy 2H-I is that the source keeps sending copies of a packet until the destination receives successfully. A timestamp is attached to each packet. All nodes and gateways in the network can perform as relays. When there is a transmission opportunity between two relays, if one has copies in buffer which the other does not have, the copy with the earliest timestamp is delivered. If neither of them carries copies of new packets, one of them is selected as a source randomly and a new generated packet is delivered.

For the optimal average transmission delay in such mobile hybrid wireless networks, we have

Lemma 4. *The upper bound of the optimal average transmission delay $E(T)$ for a single packet in i.i.d mobility is $2\log_2(\frac{1}{1-e^{-\frac{K}{n}}} + 1)$. When $K = \Theta(n)$, $E(T) = \Theta(1)$.*

Proof. Considering in Strategy MH-M, if there is only one packet being transmitted in the network, the number of mobile nodes carrying it or its copies is $2^t - 1$ in the beginning of the t-th slot. We denote the time it takes for the gateways to get copies of the packet is $T_{s\to g}$. For convenience of calculation, we make the same assumption as in Lemma 3 that the gateways only receive copies from mobile relays directly. We have

$$\begin{aligned}&\Pr(T_{s\to g} < t)\\&= 1 - \Big(1 - \frac{1}{n}\Big)^{1\cdot K\cdot(t-1)} \cdot \ldots \cdot \Big(1 - \frac{1}{n}\Big)^{2^{t-2}\cdot K\cdot 1}\\&= 1 - \Big(1 - \frac{1}{n}\Big)^{\sum_{v=0}^{t-2} 2^v\cdot K\cdot(t-v-1)}.\end{aligned} \quad (11)$$

Because

$$\sum_{v=0}^{t-2} 2^v \cdot (t - v - 1) = (t - 1) \cdot (2^{t-1} - 1) - \sum_{v=0}^{t-2} v \cdot 2^v,$$

$$\begin{aligned}\sum_{v=0}^{t} v \cdot x^v &= x \cdot \sum_{v=0}^{t} v \cdot x^{v-1} = x \cdot \sum_{v=0}^{t} (x^v)'\\&= x \cdot \Big(\sum_{v=0}^{t} x^v\Big)' = x \cdot \Big(\frac{1 - x^{t+1}}{1 - x}\Big)'\\&= x \cdot \frac{1 - x^{t+1} - (t + 1) \cdot x^t \cdot (1 - x)}{(1 - x)^2},\end{aligned}$$

$$\begin{aligned}&\sum_{v=0}^{t-2} 2^v \cdot (t - v - 1)\\&\quad= (t - 1) \cdot (2^{t-1} - 1) - 2((t - 3) \cdot 2^{t-2} + 1)\\&\quad= 2^t - t - 1,\end{aligned}$$

$$\begin{aligned}\Pr(T_{s\to g} \leqslant t) &= 1 - \Big(1 - \frac{1}{n}\Big)^{(2^{t+1}-t-2)\cdot K}\\&\geqslant 1 - \Big(1 - \frac{1}{n}\Big)^{(2^t-1)\cdot K}.\end{aligned}$$

We consider two random variables T' and Y, and $Y = 2^{T'} - 1$. When n approaches infinity, if

$$\Pr(T' \leqslant t) = 1 - \Big(1 - \frac{1}{n}\Big)^{(2^t-1)\cdot K},$$

$$\Pr(Y \leqslant t) = 1 - \Big(1 - \frac{1}{n}\Big)^{t\cdot K},$$

$$E(Y) = \frac{1}{1 - \Big(1 - \frac{1}{n}\Big)^K} = \frac{1}{1 - e^{-\frac{K}{n}}},$$

$$\begin{aligned}E(T') &= E(\log_2(Y + 1)) = \sum_{j=1}^{\infty} \log_2(Y_j + 1) \cdot p_j\\&\leqslant \log_2 \sum_{j=1}^{\infty} (Y_j + 1) \cdot p_j = \log_2(E(Y) + 1).\end{aligned}$$

Because $\Pr(T_{s\to g} \leqslant t) \geqslant \Pr(T' \leqslant t)$,

$$E(T_{s\to g}) < E(T') \leqslant \log_2 \Big(\frac{1}{1 - e^{-\frac{K}{n}}} + 1\Big).$$

If we regard the time when gateways get a copy of the packet as the starting point, it can be easily proved that the expected remaining time it takes for the destination to get a copy is less than $E(T_{s\to g})$. So, the upper bound of the expected delay is $2E(T_{s\to g})$. □

Different with Strategy 2H-M, when a relay node gets a copy from one source, it may send replications to other mobile nodes. If it is carrying copies for different S-D pairs, it has to choose one pair to serve when there is a communication opportunity. Transmissions of one pair will influence the other pairs. We have to regard the whole network as a single server, and the arrival of new packets can be regarded as Poisson distributed with rate $n \cdot \lambda(n)$. According to Lemma 4, the expected service time is bounded above by $2\log_2(\frac{1}{1-e^{-\frac{K}{n}}}+1)$. According to Lemma 2 in [12], we get that when $n \cdot \lambda(n) < \frac{1}{2\log_2(\frac{1}{1-e^{-\frac{K}{n}}}+1)} < \frac{1}{E(T)}$, $D(n) = E(T)$. Thus, we have

Theorem 3. *In i.i.d mobility model, the optimal average packet delay* $D(n) \leqslant 2\log_2(\frac{1}{1-e^{-\frac{K}{n}}}+1)$ *can be achieved when* $\lambda(n) < \frac{1}{2\cdot n\cdot \log_2(\frac{1}{1-e^{-\frac{K}{n}}}+1)}$.

Theorems 1 and 2 show how average packet delay scales under representative network capacity. Theorem 3 demonstrates how capacity scales when the delay is optimal. In the following parts, we give two quantitative results describing the relationship between these two metrics in mobile hybrid wireless networks.

Theorem 4. *In i.i.d mobility model,* $\frac{D(n)}{\lambda(n)} > \frac{n}{K\cdot W}$.

Proof. For every packet from source s to destination u, we assume the average number of mobile nodes having its copies is C_s^u. C_s^u is larger than 1 when multihop routing is used. Because it is assumed that in every slot at most W packets can be transmitted between every pair of nodes, we have

$$\lambda(n) \cdot \sum C_s^u \leqslant n \cdot W.$$

Considering a packet sent from s to u, we use $E(T_g^{s,u})$ to represent its expected transmission delay when gateways are used as relays, $E(T_u^{s,u})$ to represent the delay when gateways are not used, and $E(T^{s,u})$ to represent its expected delay. When n approaches infinity,

$$\begin{aligned} E(T_g^{s,u}) &> \frac{1}{1-\left(1-\frac{1}{n}\right)^{K\cdot C_s^u}} \approx \frac{1}{1-e^{-\frac{K\cdot C_s^u}{n}}} \\ &> \frac{n}{K\cdot C_s^u}, \\ E(T_u^{s,u}) &> \frac{1}{1-\left(1-\frac{1}{n}\right)^{C_s^u}} \approx \frac{1}{1-e^{-\frac{C_s^u}{s}}} > \frac{n}{C_s^u}, \\ E(T^{s,u}) &= p_g \cdot E(T_g^{s,u}) + (1-p_g)\cdot E(T_u^{s,u}) \\ &> \frac{n}{K\cdot C_s^u}, \end{aligned}$$

where p_g is the probability that a packet is sent through gateways. According to the argument in the proof of Lemma 2, $p_g = \Theta(\frac{K}{n\cdot\lambda})$. Thus we have

$$\begin{aligned} D(n) &= \frac{1}{n}\cdot\sum T^{s,u} > \frac{1}{n}\cdot\sum\frac{n}{K\cdot C_s^u} \\ &= \frac{n}{K}\cdot\frac{1}{n}\cdot\sum\frac{1}{C_s^u}. \end{aligned}$$

According to Jensen's inequality,

$$D(n) > \frac{n}{K}\cdot\frac{1}{\frac{1}{n}\cdot\sum C_s^u} > \frac{n}{K}\cdot\frac{\lambda(n)}{W},$$

$$\frac{D(n)}{\lambda(n)} > \frac{n}{K\cdot W}. \qquad \square$$

We denote the traditional wireless networks without gateways as *flat wireless networks*. Capacity can achieve $O(\frac{1}{\sqrt{n}})$ in such flat wireless networks[9]. According to [37], capacity of static hybrid wireless networks

$$\lambda(n) = \begin{cases} O\Big(\frac{W\cdot K}{n}\Big), & \text{if } K = \Omega(\sqrt{\frac{n}{\log n}}), \\ O\Big(\frac{W}{\sqrt{n\cdot\log n}}\Big), & \text{if } K = O(\sqrt{\frac{n}{\log n}}). \end{cases} \tag{12}$$

$K = O(\sqrt{n})$ can be regarded as the *critical number* of gateways. It is the threshold above which capacity of hybrid wireless network is better than that of flat wireless networks. In mobile hybrid wireless networks, because mobility can increase capacity[10], there are two interesting questions. The first is when $K = O(\sqrt{n})$, what is the delay above which the capacity is better than that of static wireless networks? From Theorem 2, we get that when $K = O(\sqrt{n})$, $\Omega(\sqrt{n})$ is a lower bound of the delay that has to be tolerated in order to guarantee the capacity is $\omega(\frac{1}{\sqrt{n}})$. The other question is, what is the critical average delay above which capacity of mobile hybrid wireless networks is better than that of static hybrid wireless networks with the same number of gateways? We have the following theorem.

Theorem 5. *In i.i.d mobility model, compared with static hybrid wireless networks, when* $K = O(\sqrt{n})$, *the critical average delay of mobile hybrid wireless networks is* $\Theta(\sqrt{n})$. *When* $\omega(\sqrt{n}) \leqslant K < n$, *the critical average delay is* $\Theta(\frac{K^2}{n})$.

Proof. The first part of the theorem is easy to prove. We focus on the proof of the second part.

Capacity of the mobile hybrid wireless networks constitutes of two parts. One is that achieved by transmissions through gateways, and the other is that without gateways. The former part is bounded above by $\frac{W\cdot K}{n}$.

Compared with (12), capacity of mobile hybrid wireless networks outperforms that of static hybrid wireless networks iff the latter is $\omega(\frac{W\cdot K}{n})$. Thus, the critical average delay is that above which the capacity of mobile wireless networks is $\omega(\frac{W\cdot K}{n})$.

According to [17], capacity of wireless networks is bounded above by $\frac{4W}{\delta\cdot\overline{d}\cdot\sqrt{\pi\cdot n}}$, where $\overline{d}$ is the lower bound of the average distance packets are relayed by wireless transmission. In order to guarantee the capacity is $\omega(\frac{W\cdot K}{n})$, $\overline{d}$ should be $o(\frac{\sqrt{n}}{K})$.

In i.i.d mobility model, for a pair of nodes, the probability that one falls into the disk with radius $r^* = \frac{n^{\frac{1}{2}-\beta}}{K}(\beta > 0)$ centered at the other node in t slots is $1-(1-\pi(r^*)^2)^t \approx 1-e^{-\pi(r^*)^2\cdot t}$.

The probability approaches to 1 only when $t > \frac{K^2\cdot n^{2\beta}}{\pi n} = \omega(\frac{K^2}{n})$.

Considering the probability $(1-\pi(r^*)^2)^t = e^{-\pi(r^*)^2\cdot t}$, it approaches to 1 when $t = o(\frac{K^2}{n})$.

For the probability to establish a path between an S-D pair (direct or multi-hop) whose total length is $o(r^*)$ in $o(\frac{K^2}{n})$ time, we get that it approaches to 0 as $n \to \infty$. Thus, the capacity of mobile hybrid wireless networks is not better than that of the static hybrid wireless networks when $D(n) = \Theta(\frac{K^2}{n})$. □

It should be noticed that in the proof of Theorems 1∼4, we make the assumption that $r(n) = \frac{1}{\sqrt{\pi n}}$ for the convenience of analysis. In the proof of Theorem 5, this assumption is not used. In fact it is assumed that $r(n) \geqslant \Theta(\frac{\sqrt{n}}{K})(\omega(\sqrt{n}) \leqslant K < n)$. This is because that the critical average delay is the lower bound of the delay needed for the network capacity to cross a threshold. It equals to the least waiting time for two randomly distributed nodes to communicate successfully in the distance of $\Theta(\frac{\sqrt{n}}{K})$.

4.2 Random Walk Mobility

Lemma 5. *For two nodes following Random Walk Mobility model with constant speed, the expected value of their first meeting time is* $\frac{1}{\frac{8S}{\pi}\cdot r(n)}$, *the variance is* $\frac{1-\frac{8S}{\pi}\cdot r(n)}{(\frac{8S}{\pi}\cdot r(n))^2}$.

Proof. As shown in Fig.2, we take two nodes a and b into consideration, and assume that they start moving at the same time. Their positions in the end of the t-th time slot are $C_a(t)$ and $C_b(t)$. $\|C_a(0)-C_b(0)\| = r$ and $\|C_a(0^-)-C_b(0^-)\| < r$, where $\|\cdot\|$ represents the Euclidean distance in a 2D space. In the beginning of the t-th time slot, each node chooses a random direction uniformly from $[0, 2\pi]$, and keeps moving with speed S for a unit slot. Because the square is assumed to be a torus, the nodes will not bounce off when they reach the boundary.

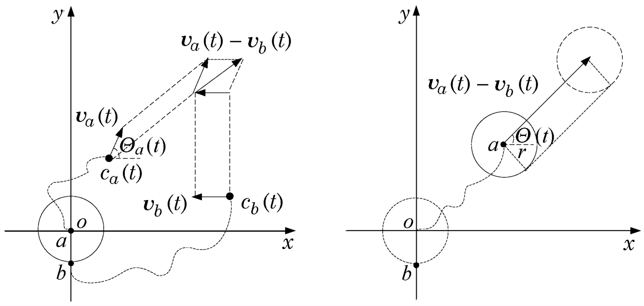

Fig.2. Two nodes a and b follow random walk mobility model, with velocity $\boldsymbol{v}_a(t)$ and $\boldsymbol{v}_b(t)$ at the t-th time slot individually. The expected first-meeting time between them equals that when one node is static and the other moves with velocity $\boldsymbol{v}_a(t)-\boldsymbol{v}_b(t)$.

If velocity of the two nodes is $\boldsymbol{v}_a(t)$ and $\boldsymbol{v}_b(t)$ in the t-th time slot, their movement is equivalent to the one in which a moves with velocity $\boldsymbol{v}_a(t)-\boldsymbol{v}_b(t)$ while b is static, as shown in Fig.3. If $\boldsymbol{v}_a(t) = S\cdot e^{i\cdot\theta_a(t)}$ and $\boldsymbol{v}_b(t) = S\cdot e^{i\cdot\theta_b(t)}$, then the angle $\theta(t)$ of the relative velocity is decided by $\theta_a(t)$ and $\theta_b(t)$. Because they are two uniformly distributed variables, it can be easily proved that $\theta(t)$ is uniformly distributed in $[0, 2\pi]$, too.

$$\|\boldsymbol{v}_a(t)-\boldsymbol{v}_b(t)\| = 2S\cdot\sin\Big(\frac{|\,\theta_a(t)-\theta_b(t)\,|}{2}\Big).$$

$$\Pr(\|\boldsymbol{v}_a(t)-\boldsymbol{v}_b(t)\| \leqslant Y) = \frac{2}{\pi}\cdot\arcsin\frac{Y}{2S},$$

where $0 \leqslant Y \leqslant 2S$, and

$$E(y_t) = \int_0^{2S^-} y_t\cdot p(y_t)\mathrm{d}y_t = \frac{4S}{\pi}.$$

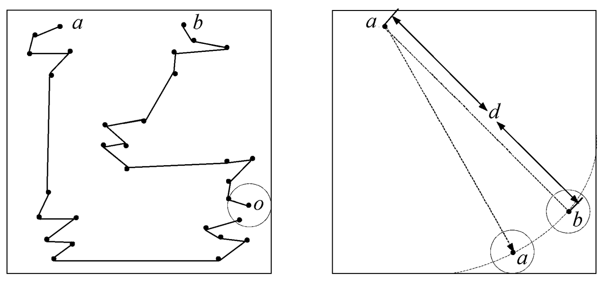

Fig.3. Two nodes a and b follow Lévy flight mobility model, and they can communicate at the position o. When their distance is d, a has to travel up to a relative distance $d \pm r(n)$ to meet b.

Because initially the nodes can be regarded as uniformly distributed, the probability that they meet for the first time in slot t can be calculated as

$$\Pr(T_f^s = t) = \Big(1-\frac{8S}{\pi}\cdot r(n)\Big)^{t-1}\cdot\frac{8S}{\pi}\cdot r(n),$$

$$E(T_f^s) = \frac{1}{\frac{8S}{\pi} \cdot r(n)}, D(T_f^s) = \frac{1 - \frac{8S}{\pi} \cdot r(n)}{\left(\frac{8S}{\pi} \cdot r(n)\right)^2}. \quad \square$$

Because there are K gateways uniformly distributed in the square, the probability that a node has met a gateway in t slots is

$$\Pr(T_f^g = t) = 1 - (1 - 2S \cdot r(n))^{K \cdot t}.$$

Lemma 6. *The expected value of first-meeting time between a mobile node and the gateways is* $\frac{1}{1-(1-2S\cdot r(n))^K}$*, and the variance is* $\frac{(1-2S\cdot r(n))^K}{(1-(1-2S\cdot r(n))^K)^2}$.

When $r(n) = \frac{1}{\sqrt{\pi n}}$, the expected time duration for a mobile node to stay within the transmission range of another node (contact time) is bounded above by $\Theta(\frac{1}{\sqrt{n}\cdot S})$. Because transmission rate of the nodes is $\Theta(1)$, S should be $\Theta(\frac{1}{\sqrt{n}})$ in order to guarantee that at least a packet can be delivered when two nodes contact. In this case, the first-meeting time between two mobile nodes is the same as that under i.i.d mobility model in scale sense, so is that between a mobile node and gateways. Thus, under the random walk mobility model with speed $\Theta(\frac{1}{\sqrt{n}})$, delay and capacity can achieve the same trade-offs in scale sense with those under the i.i.d mobility model.

4.3 Lévy Flight Mobility Model

Besides the previous two synthetic mobility models which are most used in simulations and theoretical analysis, we consider Lévy flight mobility which can model human mobility. Fig.3 shows the movement of two nodes following Lévy flight mobility model. Because the probability that a node moves with speed larger than $\Theta(\frac{1}{\sqrt{n}})$ is not 0 in this model, the contact time between two nodes may approaches to 0 when they move. Thus we assume that all nodes and gateways only communicate when they are static. In other words, nodes move in odd slots and communicate in even slots.

In order to derive the first-meeting time between two nodes, we need to get the time needed for a node to travel up to $\Theta(1)$ distance. For Lévy flight in unit network model, [28] gives its lower bound. We present its upper bound through the lemma below.

Lemma 7. *Time needed for a node to travel up to* $\Theta(1)$ *distance under approximated Lévy flight mobility with parameter* α *scales as* $O(n^{\frac{\alpha}{2}+\epsilon})(\epsilon > 0)$ *for unit network model.*

Proof. Using the similar notations with [28], we denote the positions of a node after its t-th movement as $\boldsymbol{Y}_\alpha(t) = (Y_\alpha^x(t), Y_\alpha^y(t))$, and $\boldsymbol{Y}_\alpha(0) = (0, 0)$. Let

$$E_\alpha(d) \triangleq \inf\{t \geqslant 0 : d(\boldsymbol{Y}_\alpha(t), \boldsymbol{Y}_\alpha(0)) \geqslant d\},$$

$$\Pr(d(\boldsymbol{Y}_\alpha(t), \boldsymbol{Y}_\alpha(t)) \geqslant d) = p_e^d(t).$$

$$\begin{aligned}\Pr(E_\alpha(d) \leqslant T) &= 1 - (1 - p_e^d(1)) \cdots (1 - p_e^d(T)) \\ &\geqslant 1 - \Big(\frac{1 - p_e^d(1) + \cdots + 1 - p_e^d(T)}{T}\Big)^T \\ &= 1 - \Big(1 - \frac{\sum_{t=1}^T p_e^d(t)}{T}\Big)^T \geqslant 1 - e^{-\sum_{t=1}^T p_e^d(t)} \\ &\geqslant 1 - e^{-T \cdot p_e^d(1)}.\end{aligned}$$

Because $\Pr(d(\boldsymbol{Y}_\alpha(t), \boldsymbol{Y}_\alpha(0)) \geqslant d) > \Pr(| Y_\alpha^x(t) | \geqslant \frac{d}{\sqrt{2}})$, thus

$$\Pr(E_\alpha(d) \leqslant T) \geqslant 1 - e^{-T \cdot \Pr(|Y_\alpha^x(1)| \geqslant \frac{d}{\sqrt{2}})}.$$

We use the same assumption as [28] that $Y_\alpha^x(t) = \sum_{i=1}^t Z_\alpha(i)$, and $Z_\alpha(t)$ are i.i.d distributed taking values in $[-1, 1]$ with the PDF $f_{Z,\alpha}(x) = \frac{\alpha}{2(n^{\frac{\alpha}{2}} - 1)} x^{-1-\alpha}$. We get that $\Pr(| Y_\alpha^x(1) | \geqslant \frac{d}{\sqrt{2}}) = \Theta(n^{-\frac{\alpha}{2}})$ when $d = \Theta(1)$. When $T = n^{\frac{\alpha}{2}+\epsilon}$ and $\epsilon > 0$, we have

$$\lim_{n \to \infty} \Pr(E_\alpha(d) \leqslant T) \geqslant 1. \quad \square$$

Based on the above lemma, we get the following result for the first-meeting time between two nodes under Lévy flight mobility.

Lemma 8. *For two nodes uniformly distributed in the square, if they follow Lévy flight mobility model with parameter* α *and their transmission range is* $r(n)$*, the expected time it takes for them to meet is bounded below by* $\Omega(\frac{n^{\frac{\alpha}{2}}}{r(n)})$*, and bounded above by* $O(\frac{n^{\frac{\alpha}{2}+\epsilon}}{r(n)})(\epsilon > 0)$.

Proof. We denote the two nodes as a and b. Because they are uniformly distributed, the average distance between them is $\Theta(1)$. If a and b move with speed $\boldsymbol{v}_a(t)$ and $\boldsymbol{v}_b(t)$ individually in slot t, it equals that a moves with speed $\boldsymbol{v}_a(t)$ in slot t and $-\boldsymbol{v}_b(t)$ in slot $t+1$, while b is static. In Lévy flight mobility model, according to [28], the critical delay for the unit network model is $\Omega(n^{\frac{\alpha}{2}})$. This means that, it will take $\Omega(n^{\frac{\alpha}{2}})$ time for a node to move up to the total distance $d = \Theta(1)$. When a finishes the d-distance movement, the possible transmission range of a forms a ring with width $r(n)$ (or parts of a ring because of the effect of boundary). The probability that b can communicate with a is $\Theta(\frac{r(n)^2}{d\cdot r(n)}) = \Theta(r(n))$. The number of such a $\Theta(1)$-movement needed for a to meet b can be regarded as geometric distributed. Thus, the expected time needed for them to meet is $\Omega(\frac{n^{\frac{\alpha}{2}}}{r(n)})$.

According to Lemma 7 and following a similar proof, we get that the expected time is $O(\frac{n^{\frac{\alpha}{2}+\epsilon}}{r(n)})(\epsilon > 0)$. $\square$

Recalling the proof of Theorem 5, when $\omega(\sqrt{n}) \leqslant K < n$, the critical average delay equals that needed for a node to move into the disk with radius $O(\frac{\sqrt{n}}{K})$

centered at another randomly selected node. According to Lemma 8, we have

Theorem 6. *In Lévy flight mobility model, compared with static hybrid wireless networks, when* $\omega(\sqrt{n}) \leqslant K < n$*, the critical average delay is* $\Theta(n^{\frac{\alpha-1}{2}} \cdot K)$.

Next we investigate the bound of average packet delay divided by the capacity. It is necessary to calculate the time for a node to meet the gateways firstly. With larger α, it is more likely for a node to visit the places nearby. For K gateways uniformly distributed in the square, the probabilities that the node meets different gateways are different. We cannot simply divide the expected first-meeting time between two nodes by K to get the expected first-meeting time between a node and the gateways.

Lemma 9. *If* $K = O(n^{\eta})$ $(0 \leqslant \eta < 1)$ *gateways are uniformly distributed in the square, a node with transmission range* $r(n) = \Theta(\frac{1}{\sqrt{n}})$ *moves under Lévy flight mobility model will meet a gateway in the expected time* $[\Omega(n^{\frac{\alpha+1-3\eta}{2}}), O(n^{\frac{(1-\eta)\cdot(\alpha+1)}{2}} \ln n)]$.

Proof. Considering a disk centered at the node with radius r^*, the probability that there is a gateway in it is

$$1 - (1 - \pi \cdot r^{*2})^K > 1 - e^{-\pi \cdot r^{*2} \cdot K} = 1,$$

$$\text{if} \quad r^* = \omega\Big(\frac{1}{\sqrt{K}}\Big).$$

Thus, the distance between the node and the nearest gateway is bounded above by $\Theta(\frac{1}{\sqrt{K}})$.

Following a proof similar to that of Lemma 7 in [28], we get that the time needed for a node to travel up to a distance of $\Theta(\frac{1}{\sqrt{K}})$ is $\Omega(\frac{n^{\frac{\alpha}{2}}}{K})$. Following a proof similar to that of Lemma 7, we get that the time is bounded above by $n^{\frac{(1-\eta)\cdot\alpha}{2}} \ln n$. Thus, the expected time for the node to meet a gateway is bounded below by $\Omega(\frac{n^{\frac{\alpha}{2}}}{K} \cdot \frac{1}{r(n)\cdot\sqrt{K}}) = \Omega(n^{\frac{\alpha+1-3\eta}{2}})$, and bounded above by $O(n^{\frac{(1-\eta)\cdot(\alpha+1)}{2}} \ln n)$. □

We assume that under a particular scheme there are C_s^u relaying nodes for the transmission of a packet from node s to u. It takes $O(\frac{n^{\frac{(1-\eta)\cdot(\alpha+1)}{2}} \ln n}{C_s^u})$ time for at least one of the relays to contact with a gateway. Following the similar proof of Theorem 4, we have

Theorem 7. *In Lévy flight mobility model, if the number of gateways is* $K = O(n^{\eta})(0 \leqslant \eta < 1)$*, then* $\frac{D(n)}{\lambda(n)} > O(n^{\frac{(1-\eta)\cdot(\alpha+1)}{2}} \ln n)$.

5 Conclusions

In this paper, we investigate the trade-offs between delay and capacity in mobile wireless networks with infrastructure support. We consider two synthetic models i.i.d mobility model, random walk mobility model with constant speed and Lévy flight mobility model which is based on human mobility. For i.i.d mobility model and random walk mobility model with speed $\Theta(\frac{1}{\sqrt{n}})$, we give theoretical results of average packet delay when capacity is $\Theta(1)$, $\Theta(\frac{1}{\sqrt{n}})$ individually. It is found that capacity is bounded above by $\frac{1}{2\cdot n\cdot\log_2(\frac{1}{1-e^{-\frac{K}{n}}}+1)}$ when the average packet delay is optimized. The bounds of the average delay divided by capacity under the three models are established, as well as the critical average delay for the capacity comparing with that in static hybrid wireless networks. Our work provides useful theoretical insights on the performance of mobile wireless networks with infrastructure support, and will help scheduling and routing protocols design in such networks.

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