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# Collision Attack on the Full Extended MD4 and Pseudo-Preimage Attack on **RIPEMD**

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Abstract The cryptographic hash functions Extended MD4 and RIPEMD are double-branch hash functions, which consist of two parallel branches. Extended MD4 was proposed by Rivest in 1990, and RIPEMD was devised in the framework of the RIPE project (RACE Integrity Primitives Evaluation, 1988~1992). On the basis of differential analysis and meet-in-themiddle attack principle, this paper proposes a collision attack on the full Extended MD4 and a pseudo-preimage attack on the full RIPEMD respectively. The collision attack on Extended MD4 holds with a complexity of  $2^{37}$ , and a collision instance is presented. The pseudo-preimage attack on RIPEMD holds with a complexity of  $2^{125.4}$ , which optimizes the complexity order for brute-force attack. The results in this study will also be beneficial to the analysis of other double-branch hash functions such as RIPEMD-160.

collision attack, preimage attack, hash function, Extended MD4, RIPEMD Keywords

#### Introduction 1

Cryptographic hash functions remain one of the most prevailing cryptographic primitives, and they can guarantee the security of many cryptosystems and protocols such as digital signature, message authentication code, and so on. In 1990, Rivest introduced the first dedicated hash function MD4<sup>[1]</sup>. After the publication of MD4, several dedicated hash functions were proposed, and these functions are called MD-family. Depending on the methods of the message expansion and the number of parallel branches, the MD-family is divided into three subfamilies. The first subfamily is MD4-family, which consists of MD4<sup>[1]</sup>, MD5<sup>[2]</sup> and HAVAL<sup>[3]</sup>. The characteristics of MD4-family are using roundwise permutations for the message expansion and only one branch of computation. The second subfamily is RIPEMD-family, which consists of RIPEMD<sup>[4]</sup>, RIPEMD-{128, 160, 256, 320}<sup>[5]</sup> and Extended  $MD4^{(1)}$ . The crucial difference between MD4family and RIPEMD-family is that RIPEMD-family uses two parallel branches of computations. The third subfamily is SHA-family, which consists of SHA-{0, 1,  $224, 256, 384, 512\}^{[6-8]}$ . These functions use only one branch of computation, but the message expansion is achieved by some recursively defined functions. Several important breakthroughs have been made in the cryptanalysis of hash functions and they imply that most of the current standard hash functions are vulnerable. In this circumstance, National Institute of Standards and Technology (NIST) launches the NIST Hash Competition<sup>(2)</sup>, a public competition to develop a new hash standard, which is called SHA-3 and was announced at 2012.

From the security perspective, a cryptographic hash function should satisfy several properties such as preimage resistance, second-preimage resistance and collision resistance. A hash function is considered academically broken if it is possible to find a collision or (second) preimage faster than birthday attack or brute

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 $<sup>^{(1)}</sup>$ Extended MD4 has two copies of (modified) MD4, and the hash value is obtained by concatenating the results of both copies of MD4. However, its security against collision attack is much stronger than that of MD4, so we classify it as a double-branch hash <sup>(2)</sup>NIST. Cryptographic hash project, http://csrc.nist.gov/groups/ST/hash/index.html, September 2012.

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force attack respectively. The first analysis of MD4 and MD5 was made by Vaudenay<sup>[9]</sup> and by den Boer and Bosselaers<sup>[10]</sup>. Along with the development of the hash functions, there is some continuous analysis on them and the work reveals that most of the hash functions are not so secure as claimed<sup>[11-21]</sup>. Wang *et al.* presented a series collision attacks on the most prevailing ARX-type (modular addition, rotation and bitwise XOR) hash functions including MD4<sup>[22]</sup>, RIPEMD<sup>[22]</sup>, RIPEMD-128<sup>[23]</sup>, MD5<sup>[24]</sup>, SHA-0<sup>[25]</sup>, SHA-1<sup>[26]</sup>, HAVAL<sup>[27-28]</sup>, SIMD<sup>[29]</sup> and Skein<sup>[30]</sup>, etc. using an attack technique which is based on differential cryptanalysis<sup>[31]</sup>. Wang's method was also adopted in searching the secondpreimage of  $MD4^{[32]}$ , and was further developed in the analysis of SHA- $1^{[33-34]}$ . With the collision attacks on some dedicated hash functions, more attentions are payed to evaluate the preimage resistance of hash functions. So far, in the sense of preimage resistance, full MD2<sup>[35-36]</sup>, MD4<sup>[14,37-41]</sup>, MD5<sup>[42]</sup>, HAVAL<sup>[43]</sup>, Tiger<sup>[39]</sup>, and step-reduced RIPEMD<sup>[44-45]</sup>, RIPEMD-128<sup>[46]</sup>, RIPEMD-160<sup>[46]</sup>, SHA-0<sup>[47]</sup>, SHA-1<sup>[47]</sup>, SHA-2<sup>[39,48]</sup>,  $\text{GOST}^{[49]}$ , Skein<sup>[50]</sup>, Whirlpool<sup>[51]</sup>, Grøstl<sup>[52]</sup>, etc. have been broken. Most of these preimage attacks follow a technique of meet-in-the-middle[53-54]. It is worth noting that, in FSE 2012, Li et al. converted meet-in-the-middle pseudo-preimage attack into pseudo collision attack, and gave applications to SHA-2.  $etc^{[55]}$ .

Extended MD4 was proposed in the original article<sup>[1]</sup> by Rivest in 1990 with 256-bit hash value. Its compression function consists of two parallel branches called left branch and right branch. The left branch is the standard MD4 algorithm, and the right branch is a modified MD4 algorithm. The initial values of left branch and right branch are denoted by  $IV_0$  and  $IV_1$  ( $IV_0 \neq IV_1$ ) respectively. Dobbertin proposed a pseudo-collision attack for Extended MD4 with a complexity of  $2^{40}$  under the condition that  $IV_0 = IV_1$  is prescribed. However, no collision attack on Extended MD4 under the standard initial values was proposed yet. RIPEMD<sup>[4]</sup> was developed in the European RIPE project (RACE Integrity Primitives Evaluation, 1988~1992), and was designed by Dobbertin, Bosselaers, and Preneel. Its compression function consists of two parallel branches of transformations and generates the output by mixing the results of the two branches. Wang et al. presented a collision attack on  $RIPEMD^{[22]}$ . As for the preimage attack, the security of step-reduced RIPEMD has been analyzed in  $[44-45^{(3)}, 56]$ .

Many studies have been conducted on the security of ARX-type hash functions using Wang's method. How-

ever, owing to the two parallel branches of RIPEMDfamily, it is difficult to deduce the correct differential path and to use message modification technique to improve the success probability for RIPEMD-family. The security of RIPEMD-family hash functions against collision attack has been strengthened greatly. [22] reports that among 30 selected collision differential paths, only one can produce a real collision, and in other paths, the conditions of both branches in some step cannot hold simultaneously. Extended MD4 is such a representative hash function of RIPEMD-family. It is difficult to deduce a correct differential path for both branches and to modify the messages to greatly improve the success probability of the attack, so to find a practical collision. In Section 3, we propose a practical collision attack on Extended MD4 under the standard initial values, and present a collision instance for Extended MD4. By choosing a proper message difference, we can find a differential path of both left branch and right branch, and deduce the corresponding sufficient conditions that ensure the differential path hold. We use the message modification techniques to modify the messages so that almost all sufficient conditions hold. Our attack requires less than  $2^{37}$  computations to get a collision of the full Extended MD4. To the best of our knowledge, this is the first work that a practical collision attack on the full Extended MD4 has been proposed.

The meet-in-the-middle technique works efficiently on narrow-pipe Merkle-Damgård hash functions. However, because the size of the internal state of RIPEMD is 256 bits which is a double of the size of hash value, if we apply the meet-in-the-middle technique directly, it will has no advantage compared with the complexity  $2^{128}$  of the brute force attack. So it is difficult to apply the meet-in-the-middle technique to propose preimage attacks on RIPEMD directly. In Sections 4 and 5, we find some new observations and propose the first pseudo-preimage attack on the full RIPEMD using the meet-in-the-middle principle combined with some other techniques such as initial structure, partial-matching, partial-fixing. The complexity of our attack to find a pseudo-preimage of RIPEMD is  $2^{125.4}$ . The attack optimizes the complexity order for brute-force attack. See Table 1 for a summary of our results in Sections 4 and 5 and the comparison with the previous attacks.

The rest of the paper is organized as follows. Section 2 introduces some notations, describes the Extended MD4 and RIPEMD algorithms, and summarizes some useful properties of the Boolean functions in two hash functions. The next section proposes the detailed description of the collision attack on Extended MD4.

<sup>&</sup>lt;sup>(3)</sup>The attack contains a flaw on the message-word order. If the correct order is used, the attack can work on the first 31 steps instead of 26 steps.

Number of	Pseudo-Preimage	(Second) Preimage	Reference
Steps	Attacks	Attacks	
	Complexity	Complexity	
	Time/Memory	Time/Memory	
$26/29^*$	$2^{110}/2^{33}$	$2^{115.2}/2^{33}$	[45]
33	$2^{121}/2^{10}$	$2^{125.5}/2^{10}$	[44]
$35^{*}$	$2^{96}/2^{35}$	$2^{113}/2^{35}$	[44]
$47^{\star}$	$2^{119}/2^{10.5}$	$2^{124.5}/2^{10.5}$	[56]
48	$2^{125.4}/2^{58}$		Ours

 
 Table 1. Comparison of Our Results with Previous (Pseudo-)Preimage Attacks

Note: \*: the attacked steps start from some intermediate step.  $\star$ : the attack is only applicable to find second preimages.

Sections 4 and 5 describe the procedure of our pseudopreimage attack on the full RIPEMD compression function. Finally, Section 6 concludes the paper.

#### 2 Preliminary

#### 2.1 Notations

In order to describe our analysis conveniently, we introduce some notations, where  $0 \leq j \leq 31$ .

1)  $M = (m_0, m_1, \ldots, m_{15})$  represents a 512-bit block, where  $m_i \ (0 \leq i \leq 15)$  is a 32-bit word.

2)  $\neg$ ,  $\land$ ,  $\oplus$ ,  $\lor$  denote bitwise complement, AND, XOR and OR respectively.

3)  $\ll s \gg s$  is circular shift s-bit positions to the left (right).

4)  $x \parallel y$  denotes concatenation of the two bit strings x and y.

5) +, - denote addition and subtraction modulo  $2^{32}$  respectively.

6) The least significant bit is the first bit (0th bit) and the most significant bit is the last bit (31st bit).

7)  $x_{i,j}$  denotes the *j*-th bit of 32-bit word  $x_i$ .

8)  $\Delta x_i = x'_i - x_i$  is the modular subtraction difference of two words  $x'_i$  and  $x_i$ .

9)  $x'_i = x_i[j]$  is the value obtained by modifying the *j*-th bit of  $x_i$  from 0 to 1, i.e.,  $x_{i,j} = 0$ ,  $x'_{i,j} = 1$ , and the other bits of  $x_i$  and  $x'_i$  are all equal. Similarly,  $x_i[-j]$  is the value obtained by modifying the *j*-th bit of  $x_i$  from 1 to 0.

10)  $x_i[\pm j_1, \pm j_2, \ldots, \pm j_k]$  denotes the value obtained by modifying the bits in positions  $j_1, \ldots, j_k$  of  $x_i$  according to the  $\pm$  signs.

11) In Section 3,  $(a_i, b_i, c_i, d_i)$  and  $(aa_i, bb_i, cc_i, dd_i)$  $(0 \leq i \leq 12)$  represent the chaining variables corresponding to the message block  $M_1$  of left branch and right branch respectively.

12) In Section 3,  $(a'_i, b'_i, c'_i, d'_i)$  and  $(aa'_i, bb'_i, cc'_i, dd'_i)$  $(0 \leq i \leq 12)$  represent the chaining variables corresponding to the message block  $M'_1$  of left branch and right branch respectively. 13) In Sections 4 and 5,  $(a_i, b_i, c_i, d_i)$  and  $(aa_i, bb_i, cc_i, dd_i)$   $(0 \le i \le 48)$  represent the chaining variables of left branch and right branch respectively.

14)  $0^{\alpha \sim \beta}$  means that the bits from the  $\beta$ -th bit to the  $\alpha$ -th bit are all 0, where  $\alpha > \beta$ .

15)  $w^{\alpha \sim \beta}$  mean that the bits from the  $\beta$ -th bit to the  $\alpha$ -th bit of the variable w are arbitrary bits.

16)  $[\alpha \sim \beta]$  means that the bits from the  $\beta$ -th bit to the  $\alpha$ -th bit are known, and all the other bits are unknown.

17)  $[\alpha \sim \beta, \gamma \sim \delta]$  means that the bits from the  $\beta$ -th bit to the  $\alpha$ -th bit and from the  $\delta$ -th bit to the  $\gamma$ -th bit are known, and all the other bits are unknown.

Note that the differential definition in Wang's method<sup>[24]</sup> is a kind of precise differential which uses the difference in terms of integer modular subtraction and the difference in terms of XOR. The combination of both kinds of differences gives attackers more information. For example, the output difference in step 1 of Table 2 (collision differential path of Extended MD4) is  $\Delta a_1 = a'_1 - a_1 = 2^{19}$ , for the specific differential path, we need to expand the one-bit difference in bit 19 into a three-bit differences in bits 19, 20 and 21. That is, we expand  $a_1[19]$  to  $a_1[-19, -20, 21]$ , which means the 19th and 20th bits of  $a_1$  are 1, and the 21st bit of  $a_1$  is 0, while the 19th and 20th bits of  $a'_1$  are 0, and the 21st bit of  $a'_1$  is 1.

# 2.2 Description of Extended MD4

The hash function Extended MD4 compresses a message of length less than  $2^{64}$  bits into a 256-bit hash value. Firstly, the algorithm pads any given message into a message with the length of 512-bit multiple. We do not describe the padding process here because it has little relation with our attack, and the details of the message padding can refer to [1]. Each 512-bit message block invokes a compression function of Extended MD4. The compression function takes a 256-bit chaining value and a 512-bit message block as input and outputs another 256-bit chaining value. The initial chaining value is a set of fixed constants. The compression function consists of two parallel branches named left branch and right branch. Left branch is the standard MD4 and right branch is a modified MD4. Each branch has three rounds, and the nonlinear functions in each round are as follows:

$$F(X, Y, Z) = (X \land Y) \lor (\neg X \land Z),$$
  

$$G(X, Y, Z) = (X \land Y) \lor (X \land Z) \lor (Y \land Z),$$
  

$$H(X, Y, Z) = X \oplus Y \oplus Z.$$

Here X, Y, Z are 32-bit words. The operations of the three functions are all bitwise. Each round of the compression function in each branch consists of 16 similar

Step	Chaining Variable for $M$	$m_i$	Shift	$\Delta m_i$	Step Difference	Chaining Variable for $M'$
1	$a_1$	$m_0$	3	$2^{16}$	$2^{19}$	$a_1[-19, -20, 21]$
2	$d_1$	$m_1$	7		$2^{27}$	$d_1[-27, 28]$
3	$c_1$	$m_2$	11		$2^{6}$	$c_1[-6, -7, 8]$
4	$b_1$	$m_3$	19			$b_1$
5	$a_2$	$m_4$	3		$-2^{10} + 2^{22} - 2^{30}$	$a_2[-10, 22, -30]$
6	$d_2$	$m_5$	7		$2^{2}$	$d_2[-2, -3, 4]$
7	$c_2$	$m_6$	11		$2^{17}$	$c_2[17]$
8	$b_2$	$m_7$	19		$-2^{21}$	$b_2[21, -22]$
9	$a_3$	$m_8$	3		$-2 - 2^{13}$	$a_3[1, -2, -13]$
10	$d_3$	$m_9$	7		$2^{9}$	$d_3[-9, 10]$
11	$c_3$	$m_{10}$	11		$2^{28}$	$c_{3}[28]$
12	$b_3$	$m_{11}$	19		$-2^{8}$	$b_3[8,-9]$
13	$a_4$	$m_{12}$	3		$-2^4 - 2^{16}$	$a_4[-4, -16]$
14	$d_4$	$m_{13}$	7			$d_4$
15	$c_4$	$m_{14}$	11		$2^{7}$	$c_4[-7,8]$
16	$b_4$	$m_{15}$	19			$b_4$
17	$a_5$	$m_0$	3	$2^{16}$	$-2^{7}$	$a_5[7,-8]$
18	$d_5$	$m_4$	5			$d_5$
19	$c_5$	$m_8$	9			$c_5$
20	$b_5$	$m_{12}$	13			$b_5$
21	$a_6$	$m_1$	3		$-2^{10}$	$a_6[-10]$
22	$d_6$	$m_5$	5			$d_6$
23	$c_6$	$m_9$	9			$c_6$
24	$b_6$	$m_{13}$	13			$b_6$
25	$a_7$	$m_2$	3		$-2^{13}$	$a_7[-13]$
26	$d_7$	$m_6$	5			$d_7$
27	$c_7$	$m_{10}$	9			$c_7$
28	$b_7$	$m_{14}$	13			$b_7$
29	$a_8$	$m_3$	3		$-2^{16}$	$a_8[-16]$
30	$d_8$	$m_7$	5			$d_8$
31	$c_8$	$m_{11}$	9			$c_8$
32	$b_8$	$m_{15}$	13			$b_8$
33	$a_9$	$m_0$	3	$2^{16}$		$a_9$
÷	÷	:	÷	:	÷	
48	$b_{12}$	$m_{15}$	15			$b_{12}$

 Table 2. Collision Differential Path of Extended MD4

steps, and in each step one of the four chaining variables a, b, c, d is updated.

$$\begin{split} \phi_0(a, b, c, d, m_k, s) &= (a + F(b, c, d) + m_k) \lll s, \\ \phi_1(a, b, c, d, m_k, s) &= (a + G(b, c, d) + m_k + 5a827999) \\ &\ll s, \\ \phi_2(a, b, c, d, m_k, s) &= (a + H(b, c, d) + m_k + 6ed9eba1) \\ &\ll s, \\ \psi_0(a, b, c, d, m_k, s) &= (a + F(b, c, d) + m_k) \lll s, \\ \psi_1(a, b, c, d, m_k, s) &= (a + G(b, c, d) + m_k + 50a28be6) \\ &\ll s, \\ \psi_2(a, b, c, d, m_k, s) &= (a + H(b, c, d) + m_k + 5c4dd124) \\ &\ll s. \end{split}$$

The initial value of left branch is  $(a_0, b_0, c_0, d_0) = (67452301, \text{ efcdab89}, 98\text{badcfe}, 10325476)$ . The ini-

tial value of right branch is  $(aa_0, bb_0, cc_0, dd_0) = (33221100, 77665544, bbaa9988, ffeeddcc).$ 

Compression Function of Extended MD4. For a 512-bit message block  $M = (m_0, m_1, \ldots, m_{15})$  of the padded message  $\overline{M}$ , the compression function consists of left branch and right branch.

Left Branch. For the 512-bit block M, left branch is as follows:

1) Let  $(a_0, b_0, c_0, d_0)$  be the input of left branch for M. If M is the first message block to hashed, then  $(a_0, b_0, c_0, d_0)$  are set to be the initial value. Otherwise it is the output from compressing the previous message block by left branch.

2) Perform the following 48 steps (three rounds):

For j = 0, 1, 2, For i = 0, 1, 2, 3,

 $a = \phi_j(a, b, c, d, m_{ord(j, 16j+4i+1)}, s_{j, 16j+4i+1}),$ 

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 $d = \phi_j(d, a, b, c, m_{ord(j, 16j+4i+2)}, s_{j, 16j+4i+2}),$ 

 $c = \phi_j(c, d, a, b, m_{ord(j, 16j+4i+3)}, s_{j, 16j+4i+3}),$ 

 $b = \phi_j(b, c, d, a, m_{ord(j, 16j+4i+4)}, s_{j, 16j+4i+4}),$ 

where  $m_{ord}$  means order of message.

The compressing result of left branch is  $(A, B, C, D) = (a_0 + a, b_0 + b, c_0 + c, d_0 + d).$ 

Right Branch. For the 512-bit block M, right branch is as follows:

1) Let  $(aa_0, bb_0, cc_0, dd_0)$  be the input of right branch for M. If M is the first block to be hashed,  $(aa_0, bb_0, cc_0, dd_0)$  are the initial value. Otherwise it is the output from compressing the previous message block by right branch.

2) Perform the following 48 steps (three rounds):

For j = 0, 1, 2,

For i = 0, 1, 2, 3,

$$\begin{split} &aa = \psi_j(aa, bb, cc, dd, m_{ord(j,16j+4i+1)}, s_{j,16j+4i+1}), \\ &dd = \psi_j(dd, aa, bb, cc, m_{ord(j,16j+4i+2)}, s_{j,16j+4i+2}), \\ &cc = \psi_j(cc, dd, aa, bb, m_{ord(j,16j+4i+3)}, s_{j,16j+4i+3}), \\ &bb = \psi_j(bb, cc, dd, aa, m_{ord(j,16j+4i+4)}, s_{j,16j+4i+4}). \end{split}$$

The compressing result of right branch is  $(AA, BB, CC, DD) = (aa_0 + aa, bb_0 + bb, cc_0 + cc, dd_0 + dd).$ 

Note that after every 16-word block is processed, the values of the *a* register in left branch and the *aa* register in right branch are exchanged. The ordering of message words and the details of the shift positions can be seen in Table 3.

Table 3. Order of the Message Words and ShiftPositions in Extended MD4

Step	Order of	$\mathbf{Shift}$	Step	Order of	Shift	Step	Order of	$\mathbf{Shift}$
i	Message	$s_{0,i}$	i	Message	$s_{1,i}$	i	Message	$s_{2,i}$
	ord(0,i)			ord(1,i)			ord(2,i)	
1	0	3	17	0	3	33	0	3
2	1	7	18		5	34	8	9
3	2	11	19	8	9	35	4	11
4	3	19	20	12	13	36	12	15
5	4	3	21	1	3	37	2	3
6	5	$\overline{7}$	22	5	5	38	10	9
7	6	11	23	9	9	39	6	11
8	7	19	24	13	13	40	14	15
9	8	3	25	2	3	41	1	3
10	9	$\overline{7}$	26	6	5	42	9	9
11	10	11	27	10	9	43	5	11
12	11	19	28	14	13	44	13	15
13	12	3	29	3	3	45	3	3
14	13	$\overline{7}$	30	7	5	46	11	9
15	14	11	31	11	9	47	7	11
16	15	19	32	15	13	48	15	15

If M is the last block of  $\overline{M}$ ,  $(A \parallel B \parallel C \parallel D \parallel AA \parallel BB \parallel CC \parallel DD)$  is the hash value of the message  $\overline{M}$ . Otherwise, repeat the above process with the next 512-bit message block by taking (A, B, C, D) and

(AA, BB, CC, DD) as the input chaining variables of left branch and right branch respectively.

## 2.3 Description of RIPEMD

The hash function RIPEMD compresses any arbitrary length message into a message with the length of 128 bit. Firstly RIPEMD pads any given message into a message with the length of 512 bit multiple. For each 512-bit message block, RIPEMD compresses it into a 128-bit hash value by a compression function. The compression function consists of two parallel operations, which are denoted by left branch and right branch respectively. The nonlinear functions are the same as the functions F, G, H in Extended MD4.

Left Branch. For a 512-bit block  $M = (m_0, m_1, \ldots, m_{15})$ , left branch is as follows:

1) Let  $(a_0, b_0, c_0, d_0)$  be the input of left branch for M. If M is the first block to be hashed,  $(a_0, b_0, c_0, d_0)$  is the initial value. Otherwise it is the output of the previous block compressing.

- 2) Perform the following 48 steps (three rounds):
  - a) For  $i = 1, \dots, 16$ , do the following 16 operations:  $a_i = d_{i-1}, b_i = (a_{i-1} + F(b_{i-1}, c_{i-1}, d_{i-1}) + m_{\sigma(i)}) \ll s_i, c_i = b_{i-1}, d_i = c_{i-1}.$
  - b) For  $i = 17, \dots, 32$ , do the following 16 operations:  $a_i = d_{i-1}, b_i = (a_{i-1} + G(b_{i-1}, c_{i-1}, d_{i-1}) + m_{\sigma(i)} + 5a827999) \ll s_i,$  $c_i = b_{i-1}, d_i = c_{i-1}.$
  - c) For  $i = 33, \dots, 48$ , do the following 16 operations:  $a_i = d_{i-1}, b_i = (a_{i-1} + H(b_{i-1}, c_{i-1}, d_{i-1}) + m_{\sigma(i)} + 6ed9eba1) \ll s_i,$  $c_i = b_{i-1}, d_i = c_{i-1}.$

Right Branch. For a 512-bit block  $M = (m_0, m_1, \ldots, m_{15})$ , right branch is as follows:

1) Let  $(aa_0, bb_0, cc_0, dd_0)$  be the input of right branch for M. If M is the first block to be hashed,  $(aa_0, bb_0, cc_0, dd_0)$  is the initial value. Otherwise it is the output of the previous block compressing.

2) Perform the following 48 steps (three rounds):

- a) For  $i = 1, \dots, 16$ , do the following 16 operations:  $aa_i = dd_{i-1}, \ bb_i = (aa_{i-1} + F(bb_{i-1}, cc_{i-1}, dd_{i-1}) + m_{\sigma(i)} + 50a28be6) \ll s_i, \ cc_i = bb_{i-1}, \ dd_i = cc_{i-1}.$
- b) For  $i = 17, \dots, 32$ , do the following 16 operations:  $aa_i = dd_{i-1}, \ bb_i = (aa_{i-1} + G(bb_{i-1}, cc_{i-1}, dd_{i-1}) + m_{\sigma(i)}) \ll s_i, \ cc_i = bb_{i-1}, \ dd_i = cc_{i-1}.$
- c) For  $i = 33, \dots, 48$ , do the following 16 operations:  $aa_i = dd_{i-1}, \ bb_i = (aa_{i-1} + H(bb_{i-1}, cc_{i-1}, dd_{i-1}) + m_{\sigma(i)} + 5c4dd124) \ll s_i, \ cc_i = bb_{i-1}, \ dd_i = cc_{i-1}.$

Note that the initial values of left branch and right branch are identical. The orders of message words and the details of the shift positions can be seen in Table 4. Add the output of left branch to the output of right branch as follows:  $H_0 = b_0 + c_{48} + dd_{48}, H_1 =$  $c_0+d_{48}+aa_{48}, H_2 = d_0+a_{48}+bb_{48}, H_3 = a_0+b_{48}+cc_{48}.$ If M is the last message block of the message MM, then  $H(MM) = H_0 \parallel H_1 \parallel H_2 \parallel H_3$  is the hash value for the message MM. Otherwise repeat the compression process for the next 512-bit message block and  $(H_0, H_1, H_2, H_3)$  as input. For the completed specification, refer to [4].

 Table 4. Word Processing Orders and Shift

 Positions in RIPEMD

Step	Order of	Shift	Step	Order of	Shift	Step	Order of	Shift
i	Message	$s_i$	i	Message	$s_i$	i	${\it Message}$	$s_i$
	$\sigma(i)$			$\sigma(i)$			$\sigma(i)$	
1	0	11	17	7	7	33	3	11
2	1	14	18	4	6	34	10	13
3	2	15	19	13	8	35	2	14
4	3	12	20	1	13	36	4	7
5	4	5	21	10	11	37	9	14
6	5	8	22	6	9	38	15	9
7	6	7	23	15	7	39	8	13
8	7	9	24	3	15	40	1	15
9	8	11	25	12	7	41	14	6
10	9	13	26	0	12	42	7	8
11	10	14	27	9	15	43	0	13
12	11	15	28	5	9	44	6	6
13	12	6	29	14	7	45	11	12
14	13	7	30	2	11	46	13	5
15	14	9	31	11	13	47	5	7
16	15	8	32	8	12	48	12	5

# 2.4 Some Basic Conclusions of the Three Nonlinear Functions

We will recall some well-known properties of the three nonlinear Boolean functions because they are very helpful for determining the collision differential path, the corresponding sufficient conditions and the initial structure. In the following,  $x \in \{0, 1\}$ ,  $y \in \{0, 1\}$  and  $z \in \{0, 1\}$ .

**Proposition 1.** For the nonlinear function  $F(x, y, z) = (x \land y) \lor (\neg x \land z)$ , there are the following properties.

- 1) a)  $F(x, y, z) = F(\neg x, y, z)$  if and only if y = z.
  - b) F(x, y, z) = x and  $F(\neg x, y, z) = \neg x$  if and only if y = 1 and z = 0.
  - c)  $F(x, y, z) = \neg x$  and  $F(\neg x, y, z) = x$  if and only if y = 0 and z = 1.
- 2) a)  $F(x, y, z) = F(x, \neg y, z)$  if and only if x = 0.
  - b) F(x, y, z) = y and  $F(x, \neg y, z) = \neg y$  if and only if x = 1.

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3) a) F(x, y, z) = F(x, y, ¬z) if and only if x = 1.
b) F(x, y, z) = z and F(x, y, ¬z) = ¬z if and only if x = 0.

**Proposition 2.** For the nonlinear function  $G(x, y, z) = (x \land y) \lor (x \land z) \lor (y \land z)$ , there are the following properties:

- 1) a)  $G(x, y, z) = G(\neg x, y, z)$  if and only if y = z. b) G(x, y, z) = x and  $G(\neg x, y, z) = \neg x$  if and only if  $y \neq z$ .
- 2) a)  $G(x, y, z) = G(x, \neg y, z)$  if and only if x = z. b) G(x, y, z) = y and  $G(x, \neg y, z) = \neg y$  if and only if  $x \neq z$ .
- 3) a)  $G(x, y, z) = G(x, y, \neg z)$  if and only if x = y. b) G(x, y, z) = z and  $G(x, y, \neg z) = \neg z$  if and only if  $x \neq y$ .
- 4) G(x, 0, 0) = G(0, y, 0) = G(0, 0, z) = 0.

**Proposition 3.** For the nonlinear function  $H(x, y, z) = x \oplus y \oplus z$ , there are the following properties: 1)  $H(x, y, z) = \neg H(\neg x, y, z) = \neg H(x, \neg y, z) = \neg H(x, y, \neg z)$ . 2)  $H(x, y, z) = H(\neg x, \neg y, z) = H(x, \neg y, \neg z) =$ 

2)  $H(x, y, z) = H(\neg x, \neg y, z) = H(x, \neg y, \neg z) =$  $H(\neg x, y, \neg z).$ 

**Proposition**  $4^{[56]}$ . Suppose  $(a_1, b_1, c_1, d_1)$  is known and  $m_0$  is unknown, then we can get  $b_0$ ,  $c_0$ ,  $d_0$  and  $a_0 + m_0 = (b_1 \gg 11) - F(b_0, c_0, d_0)$ . Because the input of left branch and right branch is identical, we can get  $bb_0 = b_0$ ,  $cc_0 = c_0$ ,  $dd_0 = d_0$ . According to  $bb_1 = (aa_0 + m_0 + F(bb_0, cc_0, dd_0) + 50a28be6) \ll$  $11 = (a_0 + m_0 + F(b_0, c_0, d_0) + 50a28be6) \ll 11$ , we can get  $bb_1 = ((b_1 \gg 11) + 50a28be6) \ll 11$ . Therefore,  $(aa_1, bb_1, cc_1, dd_1)$  can be obtained.

# 3 Practical Collision Attack Against Extended MD4

In this section, we present a practical collision attack on Extended MD4. Each message in the collision includes two 512-bit message blocks. We search the collision pair  $(M_0 \parallel M_1, M_0 \parallel M_1')$  in the following four parts:

1) Denote Extended MD4 by h and the hash value  $h(M_0)$  by  $(a \parallel b \parallel c \parallel d \parallel aa \parallel bb \parallel cc \parallel dd)$ . (a, b, c, d) and (aa, bb, cc, dd) are also the input chaining variables of left branch and right branch of the next compression function respectively. Find a message block  $M_0$  such that  $h(M_0)$  satisfies some conditions which are part of the sufficient conditions that ensure the differential path hold, and the conditions of  $h(M_0)$  are  $b_i = c_i(i = 19, 21), b_i = 0(i = 20, 27, 28), c_{0,20} = 1, bb_i = cc_i(i = 19, 21), bb_i = 0(i = 20, 27, 28)$  and  $cc_{0,20} = 1$ .

2) Choose an appropriate message difference  $\Delta M_1 = M'_1 - M_1$  and deduce the differential path according to the specified message difference.

3) Derive a set of sufficient conditions which ensure the differential path hold. This means that if  $h(M_1)$  satisfies all the conditions in Table 5, then  $(M_0 \parallel M_1, M_0 \parallel M_1)$  consist of a collision.

4) Modify the message  $M_1$  to fulfill most of the sufficient conditions.

Obviously the first part is easy to be carried out. We will describe the last three parts in details.

Table 5. Set of Suffcient Conditions for the DifferentialPath Given in Table 2

$c_0$	$c_{0,20} = 1$
$b_0$	$b_{0,19} = c_{0,19}, \ b_{0,20} = 0, \ b_{0,21} = c_{0,21}, \ b_{0,27} = 0,$ $b_{0,28} = 0$
$a_1$	$a_{1,19} = 1, a_{1,20} = 1, a_{1,21} = 0, a_{1,27} = 1, a_{1,28} = 1$
$d_1$	$d_{1,6} = a_{1,6}, d_{1,7} = a_{1,7}, d_{1,8} = a_{1,8}, d_{1,19} = 0, \\ d_{1,20} = 0, d_{1,21} = 0, d_{1,27} = 1, d_{1,28} = 0$
$c_1$	$c_{1,6} = 1, c_{1,7} = 1, c_{1,8} = 0, c_{1,19} = 1, c_{1,20} = 1, c_{1,21} = 1, c_{1,27} = 0, c_{1,28} = 0$
$b_1$	$b_{1,6} = 0, \ b_{1,7} = 1, \ b_{1,8} = 0, \ b_{1,10} = c_{1,10}, \ b_{1,22} = c_{1,22}, \ b_{1,27} = 0, \ b_{1,28} = 1, \ b_{1,30} = c_{1,30}$
$a_2$	$a_{2,2} = b_{1,2}, a_{2,3} = b_{1,3}, a_{2,4} = b_{1,4}, a_{2,6} = 1,$ $a_{2,7} = 1, a_{2,8} = 1, a_{2,10} = 1, a_{2,22} = 0, a_{2,30} = 1$
$d_2$	$d_{2,2} = 1, d_{2,3} = 1, d_{2,4} = 0, d_{2,10} = 0, d_{2,17} = a_{2,17}, d_{2,22} = 0, d_{2,30} = 0$
$c_2$	$c_{2,2} = 1, c_{2,3} = 0, c_{2,4} = 0, c_{2,10} = 1, c_{2,17} = 0,$ $c_{2,21} = d_{2,21}, c_{2,22} = 1, c_{2,30} = 1$
$b_2$	$b_{2,1} = c_{2,1}, b_{2,2} = 1, b_{2,3} = 1, b_{2,4} = 1, b_{2,13} = c_{2,13}, b_{2,17} = 0, b_{2,21} = 0, b_{2,22} = 1$
$a_3$	$a_{3,1} = 0, \ a_{3,2} = 1, \ a_{3,9} = b_{2,9}, \ a_{3,10} = b_{2,10}, \\ a_{3,13} = 1, \ a_{3,17} = 1, \ a_{3,21} = 0, \ a_{3,22} = 0$
$d_3$	$d_{3,1} = 0, d_{3,2} = 0, d_{3,9} = 1, d_{3,10} = 0, d_{3,13} = 0, d_{3,21} = 1, d_{3,22} = 1, d_{3,28} = a_{3,28}$
$c_3$	$c_{3,1} = 1, c_{3,2} = 1, c_{3,8} = d_{3,8}, c_{3,9} = 0, c_{3,10} = 0, c_{3,13} = 1, c_{3,28} = 0$
$b_3$	$b_{3,4} = c_{3,4}, b_{3,8} = 0, b_{3,9} = 1, b_{3,10} = 1, b_{3,16} = c_{3,16}, b_{3,28} = 0$
$a_4$	$a_{4,4} = 1, a_{4,8} = 0, a_{4,9} = 1, a_{4,16} = 1, a_{4,28} = 1$
$d_4$	$d_{4,4} = 0,  d_{4,7} = a_{4,7},  d_{4,8} = 1,  d_{4,9} = 1,  d_{4,16} = 0$
$c_4$	$c_{4,4} = 1, c_{4,7} = 1, c_{4,8} = 0, c_{4,16} = 1$
$b_4$	$b_{4,7} = d_{4,7},  b_{4,8} = d_{4,8}$
$a_5$	$a_{5,7} = 0,  a_{5,8} = 1$
$d_5$	$d_{5,7}  eq b_{4,7},  d_{5,8}  eq b_{4,8}$
$c_5$	$c_{5,7} = d_{5,7},  c_{5,8} = d_{5,8}$
$b_5$	$b_{5,10} = c_{5,10}$
$a_6$	$a_{6,10} = 1$
$d_6$	$d_{6,10} = b_{5,10}$
$c_6$	$c_{6,10} = d_{6,10}$
$b_6$	$b_{6,13} = c_{6,13}$
$a_7$	$a_{7,13} = 1$
$d_7$	$d_{7,13} = b_{6,13}$
$c_7$	$c_{7,13} = d_{7,13}$
$b_7$	$b_{7,16} = c_{7,16}$
as.	$a_{8,16} = 1$
$d_8$	$d_{8,16} = b_{7,16}$
~~o	$c_{0,10} = d_{0,10}$
~0	~0,10 ~0,10

# 3.1 Collision Differential Path for Extended MD4

Constructing the differential path and deriving the sufficient conditions go on simultaneously. On one hand, we derive the sufficient conditions according to the differential path. On the other hand, we adjust the differential path to avoid the contradictory conditions. If the sufficient conditions in some steps of left branch and right branch contradict each other, the corresponding differential path is an error and no collision can be found, then we must search other differential paths from scratch.

Almost all the conditions in the first round and some conditions in the second round can be modified to hold by the message modification technique, the other conditions in the last rounds are difficult to be modified to hold. Therefore, we will ensure the sufficient conditions in the last rounds to be as less as possible. In order to find such a differential path, we select a difference between two messages as follows:  $\Delta M_1 = M'_1 - M_1 = (\Delta m_0, \Delta m_1, \dots, \Delta m_{15})$ , where  $\Delta m_0 = 2^{16}, \Delta m_i = 0, 0 < i \leq 15$ .

The whole differential path is shown in Table 2. The first column denotes the step, the second is the chaining variable in each step for  $M_1$ , the third is the message word of  $M_1$  in each step, the fourth is the shift rotation, the fifth is the message difference between  $M_1$  and  $M'_1$ , the sixth is the chaining variable difference for  $M_1$ and  $M'_1$ , and the seventh is the chaining variable for  $M'_1$ . The empty items both in the fifth and the sixth columns denote zero differences, and steps which are not listed in the table have zero differences for message words and chaining variables.

# 3.2 Deriving the Sufficient Conditions for Differential Path

In light of the propositions of the nonlinear Boolean functions given in Subsection 2.4, we can derive the conditions that guarantee the differential path in Table 2 hold. A set of sufficient conditions is shown in Table 5.

We give an example to describe how to derive a set of sufficient conditions that guarantees the differential path in step 9 of Table 2 hold. Other conditions can be derived similarly. The differential path in step 9 of Table 2 is:

$$(a_2[-10, 22, -30], d_2[-2, -3, 4], c_2[17], b_2[21, -22]) \longrightarrow (d_2[-2, -3, 4], c_2[17], b_2[21, -22], a_3[1, -2, -13]).$$

1) According to  $a_3 = (a_2 + F(b_2, c_2, d_2) + m_8) \ll 3$ and b) of Proposition 1 1), the conditions  $c_{2,22} = 1$ and  $d_{2,22} = 0$  ensure that the change of  $b_{2,22}$  results

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in  $F(b_2[-22], c_2, d_2) - F(b_2, c_2, d_2) = -2^{22}$ , combined with  $\Delta a_2 = 2^{22}$ , which will lead to no change in  $a_3$ .

2) According to a) of Proposition 1 1), the condition  $c_{2,21} = d_{2,21}$  ensures that the change of  $b_{2,21}$  results in no change in  $a_3$ .

3) According to a) of Proposition 1 2), the condition  $b_{2,17} = 0$  ensures that the change of  $c_{2,17}$  results in no change in  $a_3$ .

4) According to a) of Proposition 1 3), the conditions  $b_{2,2} = 1$ ,  $b_{2,3} = 1$  and  $b_{2,4} = 1$  ensure that the changes in the 2nd, 3rd and 4th bits of  $d_2$  result in no change in  $a_3$ .

5) Because the shift is 3 in step 9,  $\Delta a_2 = -2^{10}$  must lead to  $\Delta a_3 = -2^{13}$ , and the condition  $a_{3,13} = 1$  results in  $a'_{13} = a_{13}[-13]$ .

6) Similarly,  $\Delta a_2 = -2^{30}$  must lead to  $\Delta a_3 = -2$ , and the condition  $a_{3,1} = 0$  and  $a_{3,2} = 1$  result in  $a'_3 = a_3[1, -2]$ .

The above 10 conditions consist of a set of sufficient conditions for the differential path in step 9.

#### 3.3 Message Modification

In order to improve the collision probability, we modify  $M_1$  so that most of the sufficient conditions in Table 5 hold. The modification includes basic and advanced techniques. Because Extended MD4 has two branches, the modification is much more complicated than that of MD4, MD5, HAVAL, etc. which only contain one branch operation.

1) We modify  $M_1$  word by word so that both branches with the modified  $M_1$  satisfy almost all the conditions in the first round.

a) By using the basic modification technique, we modify  $m_{i-1}$  such that the *i*-th step conditions in the first round of left branch hold. For example, to ensure the eight conditions of  $d_1$  in Table 5 hold, we modify  $m_1$  as follows:

 $\begin{array}{l} d_1 \longleftarrow d_1 \oplus (d_{1,19} \lll 19) \oplus (d_{1,20} \lll 20) \oplus (d_{1,21} \lll 21) \oplus (d_{1,28} \lll 28) \oplus ((d_{1,27} \oplus 1) \lll 27) \oplus ((d_{1,6} \oplus a_{1,6}) \lll 6) \oplus ((d_{1,7} \oplus a_{1,7}) \lll 7) \oplus ((d_{1,8} \oplus a_{1,8}) \lll 8), \end{array}$ 

$$m_1 \longleftarrow (d_1 \gg 7) - d_0 - F(a_1, b_0, c_0)$$

b) By using the advanced modification technique, we modify the message word from low bit to high bit to correct the corresponding conditions in the first round of right branch.

• Firstly, we can correct the conditions by bit carry. For example, because there are no constraint conditions in  $a_{1,18}$  in Table 5, we can correct  $aa_{1,19} = 0$ to  $aa_{1,19} = 1$  as follows. If  $a_{1,18} = 0$  and  $aa_{1,18} = 1$ , let  $m_0 \leftarrow m_0 + 2^{15}$ , then there is a bit carry in right branch and no bit carry in left branch, so the condition in  $aa_{1,19}$  can be corrected, and the corrected  $a_{1,19}$  will not be changed. Similarly, if  $a_{1,18} = 1$  and  $aa_{1,18} = 0$ , let  $m_0 \leftarrow m_0 - 2^{15}$ , then  $aa_{1,19}$  can be corrected and  $a_{1,19}$  will not be changed. If  $a_{1,18} = aa_{1,18}$ , we can use the lower bit carry to change  $a_{1,18}$  or  $aa_{1,18}$  such that  $a_{1,18} \neq aa_{1,18}$ , and then use the bit carry. The details for correcting  $aa_{1,19}$  are given in Table 6.

• Secondly, we can correct the condition on  $a_{i,j}$  by changing the corresponding variables in the previous steps. For example, we can correct  $dd_{1,20} = 1$  to  $dd_{1,20} = 0$  as follows. If  $b_{0,13} \oplus c_{0,13} \neq bb_{0,13} \oplus cc_{0,13}$ (which means when  $b_{0,13} = c_{0,13}$ , then  $bb_{0,13} \neq cc_{0,13}$ ; when  $b_{0,13} \neq c_{0,13}$ , then  $bb_{0,13} = cc_{0,13}$ ), let  $m_0 \leftarrow m_0 \pm 2^{10}$ , then  $a_{1,13}$  and  $aa_{1,13}$  will be changed, and the changed  $a_{1,13}, aa_{1,13}$  only cause one of  $d_{1,20}$  and  $dd_{1,20}$  to change according to 1) of Proposition 1. Then if  $d_{1,20} = dd_{1,20} = 1$ , let  $m_1 \leftarrow m_1 - 2^{13}$ , if  $d_{1,20} = dd_{1,20} = 0$ , modify the next bit of  $dd_1$ . Note that there is no condition in  $a_{1,13}$  and  $aa_{1,13}$  in Table 5, so the changes in  $a_{1,13}$  and  $aa_{1,13}$  do not invalidate the differential path. The details for correcting  $dd_{1,20}$  are given in Table 7.

2) There are  $18 \times 2 = 36$  conditions in total in the second round in both branches. We can utilize some more precise modification techniques to correct some conditions in the second round. Sometimes, it needs to add some extra conditions in the first round in advance such that the change of any condition does not affect all the corrected conditions.

**Table 6.** Message Modification for Correcting  $aa_{1,19} = 0$  to  $aa_{1,19} = 1$ 

Known Conditions	Modified $m_0$	New Chaining Variables
$\boxed{a_{1,18} = 0, aa_{1,18} = 1}$	$m_0 \longleftarrow m_0 + 2^{15}$	$aa_{1,19} = 1, a_{1,19}$ unchanged
$a_{1,18} = 1, aa_{1,18} = 0$	$m_0 \longleftarrow m_0 - 2^{15}$	$aa_{1,19} = 1, a_{1,19}$ unchanged
$a_{1,18} = aa_{1,18} = 1$	$m_0 \longleftarrow m_0 + 2^{14}$	$aa_{1,19} = 1, a_{1,19}$ unchanged
$a_{1,17} = 0, aa_{1,17} = 1$		
$a_{1,18} = aa_{1,18} = 1$	$m_0 \longleftarrow m_0 - 2^{14} - 2^{15}$	$aa_{1,19} = 1, a_{1,19}$ unchanged
$a_{1,17} = 1, aa_{1,17} = 0$		
$a_{1,18} = aa_{1,18} = 0$	$m_0 \longleftarrow m_0 + 2^{14} + 2^{15}$	$aa_{1,19} = 1, a_{1,19}$ unchanged
$a_{1,17} = 0, aa_{1,17} = 1$		
$a_{1,18} = aa_{1,18} = 0$	$m_0 \longleftarrow m_0 - 2^{14}$	$aa_{1,19} = 1, a_{1,19}$ unchanged
$a_{1,17} = 1, aa_{1,17} = 0$		

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**Table 7.** Message Modification for Correcting  $dd_{1,20} = 1$  to  $dd_{1,20} = 0$ 

Step	Case	$m_i$	Shift	Modified $m_i$	Chaining Variables	Conditions
1		$m_0$	3	$m_0 \longleftarrow m_0 \pm 2^{10}$	$a_{1,13}, aa_{1,13}$ changed	
2	Case 1	$m_1$	7	$m_1 \longleftarrow m_1 - 2^{13}$	$d_{1,20}$ changed, i.e., $d_{1,20} = 1$	$b_{0,13} \neq c_{0,13}$
					$dd_{1,20}$ unchanged, i.e., $dd_{1,20} = 1$	$bb_{0,13} = cc_{0,13}$
					$d_{1,20} = 0, dd_{1,20} = 0$	
2	Case 2	$m_2$	7		$d_{1,20}$ unchanged, i.e., $d_{1,20} = 0$	$b_{0,13} = c_{0,13}$
					$dd_{1,20}$ changed, i.e., $dd_{1,20} = 0$	$bb_{0,13} \neq cc_{0,13}$

# 3.4 Overview of the Collision Attack Algorithm

From the above description, an overview of the collision attack algorithm on Extended MD4 can be expressed as follows.

1) Find a message block  $M_0$  such that  $h(M_0) = (a \parallel b \parallel c \parallel d \parallel aa \parallel bb \parallel cc \parallel dd)$  satisfies  $b_i = c_i(i = 19, 21)$ ,  $b_i = 0(i = 20, 27, 28)$ ,  $c_{20} = 1$ ,  $bb_i = cc_i(i = 19, 21)$ ,  $bb_i = 0(i = 20, 27, 28)$  and  $cc_{20} = 1$ .

2) Repeat the following steps until we can find a message block  $M_1$  which satisfies all the sufficient conditions in the first round of left branch and right branch in Table 5.

- a) Select a random message block  $M_1$ .
- b) Modify  $M_1$  by step 1 of message modification described above.
- c) Test if the hash value of  $M_1$  satisfies all the sufficient conditions in the first round in both branches in Table 5.

3) Repeat the following steps until a collision  $(M_0 \parallel M_1, M_0 \parallel M_1')$  is found.

- a) Select random message words  $m_{14}$  and  $m_{15}$  of  $M_1$ .
- b) Modify  $M_1$  by step 1 of message modification described above such that all the conditions in  $c_4$ ,  $b_4$ ,  $cc_4$  and  $bb_4$  satisfied.
- c) Modify  $M_1$  by step 2 of message modification described above such that some conditions in the second round satisfied.

- d) Then  $M_1$  and  $M'_1 = M_1 + \Delta M_1$  satisfy all the sufficient conditions in both branches in Table 5 with the probability higher than  $2^{-36}$ .
- e) Test if the hash value of  $M_1$  is equal to the hash value of  $M'_1$ .

It is easy to find proper  $M_0$  in step 1 and to find  $M_1$ which satisfies all the sufficient conditions in the first round in both branches in Table 5, and the complexity can be neglected. There are 36 conditions in total in the second round in both branches, so  $M_1$  and  $M'_1$  lead to a collision with probability higher than  $2^{-36}$ , and the complexity to find a collision  $(M_0 \parallel M_1, M_0 \parallel M'_1)$  is less than  $2^{37}$  Extended MD4 computations. A collision for Extended MD4 can be seen in Table 8.

# 4 Strategies of the Pseudo-Preimage on the Compression Function of RIPEMD

Suppose CF is the compression function and y is a given targeting hash value, a pseudo-preimage is a pair (x, M) that satisfies CF(x, M) = y, where x is not required to be equal to the standard initial value. This section studies the pseudo-preimage resistance against the full RIPEMD. We choose proper neutral words for the first chunk and the second chunk. By constructing proper initial structure and applying the partial-fixing and partial-matching techniques, a few steps can be skipped. Combined with the exhaustive search, we can propose a pseudo-preimage on the full RIPEMD algorithm.

#### Table 8. Collision of Extended MD4

- $H_0 = b5aac7e7 \ c1664fe2 \ 01705583 \ ac3cc062 \ 65c931e6 \ 452829ae \ 527e12c7 \ 30fafffb$

- $H = 3055a689\ 7 {\rm fe}0b4a6\ 88d59251\ af8a{\rm f}d0f\ 3826bda2\ 942f0939\ c2673493\ a6c56bac$
- Note:  $H_0$  is the hash value for the message block  $M_0$  with little-endian and no message padding. H is the common hash value for the message  $M_0 \parallel M_1$  and  $M_0 \parallel M_1'$  with little-endian and no message padding.

Left	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
← First Chunk ←																
	7	4	13	1	10	6	15	3	14	0	9	5	14	2	11	8
		←	—Fir	st Ch	unk ←				Start <sub>1</sub>			→ I	nitial	Struc	ture ←	-
	3	10	2	4	9	15	8	1	14	7	0	6	11	13	5	12
	-	Start <sub>2</sub>						-	→ Secor	nd Cł	nunk -	<b>→</b>				
Right	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
							_	→ Fir	st Chunk	<b>→</b>						
	7	4	13	1	10	6	15	3	14	0	9	5	14	2	11	8
				→ Firs	st Chu	ınk—	→						$\rightarrow$	Mate	ch	
	3	10	2	4	9	15	8	1	14	7	0	6	11	13	5	12
							÷	— Exl	naustive	Searc	h ←	-				

Fig.1. Overview of pseudo-preimage attack on RIPEMD.

#### 4.1 Choice of Chunks and Neutral Words

As shown in Fig.1, we construct the initial structure in the left branch. We choose  $(m_3, b_{25})$  as the neutral words for the first chunk, and  $(m_0, b_{33})$  as the neutral words for the second chunk. The initial structure consists of eight steps in the left branch. The partial-fixing and partial-matching techniques enable us to skip seven steps in the right branch by considering two possible carried number patterns.

The first chunk is independent of  $(m_0, b_{33})$ . It starts from step 25 in the left branch with a backward computation, and ends with a forward computation until step 25 in the right branch because the initial values of both branches are identical. The second chunk is independent of  $(m_3, b_{25})$ , and it is from step 33 to step 48 in the left branch with a forward computation.

We can get partial bits of  $aa_{32}(=bb_{29})$  from the computation of the first chunk. Performing the consistency check together with the partial-matching test and exhaustive search guarantees our attack hold even if the number of matched bits is small. See Fig.2 for a pictorial depiction of the attack, where *IS* means initial structure, and *ES* means exhaustive search.

# 4.2 Details of Initial Structure, Partial-Fixing and Partial-Matching

In Table 9, the initial structure is constructed in the



Fig.2. Pseudo-preimage attack on RIPEMD.

left branch.  $(m_3, b_{25})$  are the neutral words for the first chunk, and  $(m_0, b_{33})$  are the neutral words for the second chunk. Considering the initial structure, partialfixing and partial-matching techniques, we fix the the bit positions 27 ~ 24, 19 ~ 0 of  $m_0$ , the bit positions 15 ~ 0 of  $b_{25}$ , and all the other bits of  $m_0$ ,  $b_{25}$  are free bits. All the 32 bits of  $m_3$ ,  $b_{33}$  are free bits. We try all the values of free bits in the meet-in-the-middle attack.

Let  $x_{1\text{st}}$  represents a free bit of  $m_0$  and  $y_{2\text{nd}}$  represents a free bit of  $(m_3, b_{25})$ . The free bits  $'y'_{2\text{nd}}$ s in  $m_3$  only impact  $b_{33}$  which is satisfied with probability  $2^{-32}$  in the consistency check of the initial structure. Thus in order to construct the initial structure, the remaining work is to guarantee that  $(a_{25}, b_{25}, c_{25}, d_{25})$  are independent of  $'x'_{1\text{st}}$ s, and  $(a_{33}, c_{33}, d_{33})$  are independent of  $'y'_{2\text{nd}}$ s. We choose  $a_{25} = -5a827999$ ,  $c_{25} = 0$ ,  $d_{25} = 0$  and  $b_{25} = y^{31\sim 16}_{2\text{nd}} 0^{15\sim 0}$ , which means that

i	$m_i$	$a_i$	$b_i$	$c_i$	$d_i$	Shift
25		$-k_1$	$y_{2\mathrm{nd}}^{31\sim16}0^{15\sim0}$	0	0	
26	$m_0 = x_{1st}^{31\sim28} \\ 0^{27\sim24} x_{1st}^{23\sim20} \\ 0^{19\sim0}$	0	$\begin{array}{l} m_0 \lll 12 = \\ 0^{31 \sim 12} x_{\rm 1st}^{11 \sim 8} 0^{7 \sim 4} x_{\rm 1st}^{3 \sim 0} \end{array}$	$y_{2\mathrm{nd}}^{31\sim16}0^{15\sim0}$	0	12
27	$m_9 = -k_1$	0	0	$\begin{array}{c} 0^{31\sim 12} x_{1\rm st}^{11\sim 8} \\ 0^{7\sim 4} x_{1\rm st}^{3\sim 0} \end{array}$	$y_{2\mathrm{nd}}^{31\sim16}0^{15\sim0}$	15
28	$m_5 = -k_1$	$y_{\rm 2nd}^{31\sim 16}0^{15\sim 0}$	0	0	$0^{31\sim12} x_{\rm 1st}^{11\sim8} 0^{7\sim4} x_{\rm 1st}^{3\sim0}$	9
29	$m_{14} = -k_1$	$ \begin{array}{c} 0^{31\sim 12} x_{1\rm st}^{11\sim 8} \\ 0^{7\sim 4} x_{1\rm st}^{3\sim 0} \end{array} $	$a_{28} \lll 7 = y_{2nd}^{31\sim23} 0^{22\sim7} y_{2nd}^{6\sim0}$	0	0	7
30	$m_2 = -k_1$	0	$\begin{array}{l} a_{29} \lll 11 = 0^{31 \sim 23} \\ x_{1 \mathrm{st}}^{22 \sim 19} 0^{18 \sim 15} x_{1 \mathrm{st}}^{14 \sim 11} 0^{10 \sim 0} \end{array}$	$y_{2\mathrm{nd}}^{31\sim23}0^{22\sim7}\ y_{2\mathrm{nd}}^{6\sim0}$	0	11
31	$m_{11} = -k_1$	0	0	$\begin{array}{c} 0^{31 \sim 23} x_{1\mathrm{st}}^{22 \sim 19} \\ 0^{18 \sim 15} x_{1\mathrm{st}}^{14 \sim 11} \\ 0^{10 \sim 0} \end{array}$	$y_{2\mathrm{nd}}^{31\sim23}0^{22\sim7}y_{2\mathrm{nd}}^{6\sim0}$	13
32	$m_8 = -k_1$	$y_{\rm 2nd}^{31\sim23}0^{22\sim7}y_{\rm 2nd}^{6\sim0}$	0	0	$\begin{array}{c} 0^{31 \sim 23} x_{1 \mathrm{st}}^{22 \sim 19} 0^{18 \sim 15} \\ x_{1 \mathrm{st}}^{14 \sim 11} 0^{10 \sim 0} \end{array}$	12
33	$m_3 = y_{2 n d}^{31 \sim 0}$	$\begin{array}{c} 0^{31\sim23}x_{1\mathrm{st}}^{22\sim19}0^{18\sim15}\\ x_{1\mathrm{st}}^{14\sim11}0^{10\sim0} \end{array}$	?	0	0	11
D.T.	0 11 11	1 01				

Table 9. Initial Structure

Note: ?: all possible values of  $b_{33}$ .

 $(a_{25}, b_{25}, c_{25}, d_{25})$  is independent of  $'x'_{1st}s$ . Therefore the remaining work is to guarantee that  $(a_{33}, c_{33}, d_{33})$ can be computed independently of  $'y'_{2nd}s$  in  $b_{25}$ . Let  $m_0 = x_{1st}^{31\sim 28}0^{27\sim 24}x_{1st}^{23\sim 20}0^{19\sim 0}, m_3 = y_{2nd}^{31\sim 0}, b_{25} = y_{2nd}^{31\sim 16}0^{15\sim 0}$ , we give a detailed explanation in the following Algorithm 1.

### Algorithm 1.

1) In step 26, we choose  $c_{25} = 0$  and  $d_{25} = 0$ such that the free bits  $'y'_{2nds}$  of  $b_{25}$  do not affect the variable  $b_{26}$ . Choose  $a_{25} = -5a827999$  to cancel the addition of 5a827999 for simplicity. Then the variable  $b_{26} = (m_0 + a_{25} + G(b_{25}, c_{25}, d_{25}) + 5a827999) \ll 12 =$  $(x_{1st}^{31\sim28}0^{27\sim24}x_{1st}^{23\sim20}0^{19\sim0} + (-5a827999) + G(b_{25}, 0, 0) +$  $5a827999) \ll 12 = (x_{1st}^{31\sim28}0^{27\sim24}x_{1st}^{23\sim20}0^{19\sim0}) \ll 12 =$  $0^{31\sim12}x_{1st}^{11\sim8}0^{7\sim4}x_{1st}^{3\sim0}$ , which is independent of  $'y'_{2nds}$ .

2) In step 27, we choose  $m_9 = -5a827999$  to cancel the addition of 5a827999 for simplicity. According to  $b_{27} = (m_9 + a_{26} + G(b_{26}, c_{26}, d_{26}) + 5a827999) \ll 15$ , we can get  $b_{27} = G(0^{31 \sim 12} x_{1st}^{11 \sim 8} 0^{7 \sim 4} x_{1st}^{3 \sim 0}, y_{2nd}^{31 \sim 16} 0^{15 \sim 0}, 0) \ll 15$ . According to the property of the nonlinear function G, we can get  $b_{27} = 0$ .

3) In step 28, we choose  $m_5 = -5a827999$ , and according to  $b_{28} = (m_5 + a_{27} + G(b_{27}, c_{27}, d_{27}) + 5a827999) \ll 9$ , we can get  $b_{28} = 0$ .

4) In step 29, we choose  $m_{14} = -5a827999$  to cancel the addition of 5a827999 for simplicity. According to  $b_{29} = (m_{14} + a_{28} + G(b_{28}, c_{28}, d_{28}) + 5a827999) \ll 7$ , we can get  $b_{29} = (a_{28} + G(0, 0, d_{28})) \ll 7 = a_{28} \ll 7 = y_{2nd}^{31 \sim 23} 0^{22 \sim 7} y_{2nd}^{6 \circ 0}$ .

5) In step 30, we choose  $m_2 = -5a827999$  to cancel the addition of 5a827999 for simplicity. According to

 $b_{30} = (m_2 + a_{29} + G(b_{29}, c_{29}, d_{29}) + 5a827999) \ll 11, \text{ we can}$ get  $b_{30} = (a_{29} + G(b_{29}, 0, 0)) \ll 11 = a_{29} \ll 11 = b_{26} \ll 11 = 0^{31 \sim 23} x_{1st}^{22 \sim 19} 0^{18 \sim 15} x_{1st}^{14 \sim 11} 0^{10 \sim 0}.$ 

6) In step 31, we choose  $m_{11} = -5a827999$  to cancel the addition of 5a827999 for simplicity. According to  $b_{31} = (m_{11} + a_{30} + G(b_{30}, c_{30}, d_{30}) + 5a827999) \ll 13$ , we can get  $b_{31} = G(0^{31 \sim 23} x_{1st}^{22 \sim 19} 0^{18 \sim 15} x_{1st}^{14 \sim 11} 0^{10 \sim 0}, y_{2nd}^{31 \sim 23} 0^{22 \sim 7} y_{2nd}^{6 \sim 0}, 0) \ll 13$ . According to the property of the nonlinear function G, we can get  $b_{31} = 0$ .

7) In step 32, we choose  $m_8 = -5a827999$ , and according to  $b_{32} = (m_8 + a_{31} + G(b_{31}, c_{31}, d_{31}) + 5a827999) \ll 12$ , we can get  $b_{32} = 0$ .

Thus,  $a_{33} = bb_{30} = 0^{31 \sim 23} x_{1st}^{22 \sim 19} 0^{18 \sim 15} x_{1st}^{14 \sim 11} 0^{10 \sim 0}$ ,  $c_{33} = bb_{32} = 0$ ,  $d_{33} = bb_{31} = 0$  can be computed independently of the free bits  $'y'_{2nd}$ s of  $b_{25}$ .

Owe to the bit positions  $27 \sim 24, 19 \sim 0$  of  $m_0$  are fixed, and  $(aa_{25}, bb_{25}, cc_{25}, dd_{25})$  in the right branch can be computed in the first chunk, we can obtain partial bits of  $aa_{32}$  in the forward computation in the following Algorithm 2. Table 10 illustrates the partial-matching process.

#### Algorithm 2.

1) In the forward computation for step 26 in the right branch, because  $bb_{26} = (aa_{25} + G(bb_{25}, cc_{25}, dd_{25}) + m_0) \ll$  12 and  $aa_{25}$ ,  $bb_{25}$ ,  $cc_{25}$ ,  $dd_{25}$ , the bits positions 27 ~ 24, 19 ~ 0 of  $m_0$  are fixed, we can get two candidates of bits 31 ~ 12 and 7 ~ 4 of  $bb_{26}$  for each  $m_0$  by considering two possible carried number patterns from the 23nd bit to the 24th bit.

2) In the forward computation for step 27,  $aa_{26}$ ,  $cc_{26}$ ,

<u> </u>			1.1		7.7	01.0
<i>ı</i>	$m_i$	aai	bbi	$cc_i$	$dd_i$	Shift
25		$aa_{25}$	$bb_{25}$	$cc_{25}$	$dd_{25}$	
26	$m_0[27 \sim 24, 19 \sim 0]$	$dd_{25}$	$[31 \sim 12, 7 \sim 4]$	$bb_{25}$	$cc_{25}$	12
27	$m_9$	CC25	$[31 \sim 27, 22 \sim 19, 14 \sim 0]$	$[31 \sim 12, 7 \sim 4]$	$bb_{25}$	15
28	$m_5$	$bb_{25}$	$[31 \sim 28, 8 \sim 4]$	$[31 \sim 27, 22 \sim 19, 14 \sim 0]$	$[31 \sim 12, 7 \sim 4]$	9
29	$m_{14}$	$[31 \sim 12, 7 \sim 4]$	$[14 \sim 11, 6 \sim 3]$	$[31 \sim 28, 8 \sim 4]$	$[31 \sim 27, 22 \sim 19, 14 \sim 0]$	7
30	$m_2$	$[31 \sim 27, 22 \sim 19, 14 \sim 0]$		$[14 \sim 11, 6 \sim 3]$	$[31 \sim 28, 8 \sim 4]$	11
31	$m_{11}$	$[31 \sim 28, 8 \sim 4]$			$[14 \sim 11, 6 \sim 3]$	13
32	$m_8$	$[14 \sim 11, 6 \sim 3]$				12
32	$m_8$	$aa_{32}$	$bb_{32}$	$cc_{32}$	$dd_{32}$	12
33	$m_3$	$aa_{33}$	$bb_{33}$	cc <sub>33</sub>	$dd_{33}$	11
÷	:	÷	:	:	÷	÷
48	$m_{12}$	$aa_{48}$	$bb_{48}$	$cc_{48}$	$dd_{48}$	5

Table 10. Partial Matching

 $dd_{26}$ ,  $m_9$  and bits  $31 \sim 12, 7 \sim 4$  of  $bb_{26}$  are known. According to  $bb_{27} = (aa_{26} + G(bb_{26}, cc_{26}, dd_{26}) + m_9) \ll 15$ , we can get  $2^2$  candidates of bits  $31 \sim 27, 22 \sim 19, 14 \sim 0$  of  $bb_{27}$  for each  $(m_0, bb_{26})$  by considering two possible carried number patterns from the 3rd bit to the 4th bit and two possible carried number patterns from the 11th bit to the 12th bit.

3) In the forward computation for step 28,  $aa_{27}$ ,  $dd_{27}$ ,  $m_5$ , bits 31 ~ 27,22 ~ 19 of  $bb_{27}$  and bits 31 ~ 12,7 ~ 4 of  $cc_{27}$  are known. According to  $bb_{28} = (aa_{27} + G(bb_{27}, cc_{27}, dd_{27}) + m_5) \ll 9$ , we can get 2<sup>2</sup> candidates  $bb_{28}$  of bit positions 31 ~ 28,8 ~ 4 for each  $(m_0, bb_{27}, cc_{27})$  by considering two possible carried number patterns from the 18th bit to the 19th bit and two possible carried number patterns from the 26th bit to the 27th bit.

4) In the forward computation for step 29,  $aa_{28}$ ,  $m_{14}$ , bits 31 ~ 28, 8 ~ 4 of  $bb_{28}$ , bits 31 ~ 27, 22 ~ 19, 14 ~ 0 of  $cc_{28}$  and bits 31 ~ 12, 7 ~ 4 of  $dd_{28}$  are known. According to  $bb_{29} = (aa_{28} + G(bb_{28}, cc_{28}, dd_{28}) + m_{14}) \ll 7$ , we can get 2<sup>2</sup> candidates  $bb_{29}$  of bit positions 14 ~ 11, 6 ~ 3 for each  $(m_0, bb_{28}, cc_{28}, dd_{28})$ .

5) In the backward computation from step 48 to step 33 in the right branch, by exhaustively search  $m_0$  and  $m_3$ , we can get the value ( $aa_{32}, bb_{32}, cc_{32}, dd_{32}$ ).

So far, we have successfully obtained eight bits values of  $aa_{32} = bb_{29}$  with seven unknown carried numbers in the forward computation, which is compared with the value  $aa_{32}$  computed from the backward direction.

# 5 Pseudo-Preimage Attack on the Full RIPEMD

In this section, we present the pseudo-preimage at-

tack on the full RIPEMD compression function based on the initial structure and partial-matching proposed in Section 4.

#### 5.1 Pseudo-Preimage Attack Procedure

The procedure of our pseudo-preimage attack on the full RIPEMD compression function is as follows.

1) Set chaining variables in the initial structure in the left branch as shown in Table 9. Set  $m_0$ ,  $m_2$ ,  $m_3$ ,  $m_5$ ,  $m_8$ ,  $m_9$ ,  $m_{11}$  and  $m_{14}$  as shown in Table 9.

2) Set randomly chosen values to other message words, and let  $m_{15}$  satisfy the padding.

3) For all possible values of  $m_3$  and bit positions  $31 \sim 16$  of  $b_{25}$ , in total 48 free bits, do the following:

- a) Compute  $(b_{25} \ll 7) + m_3$  for efficient consistency check. Denote this value as  $C^{1st}$ .
- b) Compute from step 25 in the backward direction in the left branch until the initial state. The values of  $b_0$ ,  $c_0$ ,  $d_0$  can be computed.  $a_0 + m_0 = (b_1 \gg 11) - F(b_0, c_0, d_0)$  can also be computed, denote this value as D.
- c) Because the initial value of the right branch is the same as the initial value of the left branch, the value of  $bb_1$  in the right branch can be computed as  $bb_1 = (a_0 + m_0 + F(b_0, c_0, d_0) + 50a28be6) \ll 11 = ((b_1 \gg 11) + 50a28be6) \ll 11$ . Therefore, we obtain the internal state at step 1 in the right branch.
- d) Compute from step 2 in the right branch in the forward direction until step 25 to get the internal state  $(aa_{25}, bb_{25}, cc_{25}, dd_{25})$ .

e) From Algorithm 2, we can obtain 8 bit positions  $14 \sim 11, 6 \sim 3$  of  $aa_{32}$ .

f) Make a table of 
$$(m_3, b_{25}, C^{1\text{st}}, aa_{32}, D, b_0, c_0, d_0).$$

4) For all possible values of bit positions  $31 \sim 28, 23 \sim 20$  of  $m_0$  and all bits of  $b_{33}$ , in total 40 free bits, do the following:

- a) Compute  $a_{33}$  as shown in Table 9.
- b) Compute  $(b_{33} \gg 11) H(c_{33}, d_{33}, a_{33}) 6ed9eba1$  for the efficient consistency check. Denote the value as  $C^{2nd}$ .
- c) Compute from step 33 until step 48 in the left branch to get the value  $(a_{48}, b_{48}, c_{48}, d_{48})$ .
- d) i) Check whether  $C^{2nd}$  is matched with  $C^{1st}$  in the table.
  - ii) If they match, compute the value  $aa_0 = a_0 = D m_0$  by the value D in the table generated in f) of step 3. From  $a_0, b_0, c_0, d_0$  in the table, we can get the initial value  $(aa_0, bb_0, cc_0, dd_0)$  of the right branch. Then compute the corresponding value of  $(aa_{48}, bb_{48}, cc_{48}, dd_{48})$  according to the given target hash value and the initial value  $(aa_0, bb_0, cc_0, dd_0)$ . Compute from step 48 to step 33 in the backward direction in the right branch to get  $aa_{32}$ . Check whether bit positions  $14 \sim 11, 6 \sim 3$  of  $aa_{32}$  are matched in the table.
  - iii) If they match, for the remaining  $(m_0, b_{33}, m_3, b_{25})$ , compute  $aa_{31}$  in the backward direction, and check whether bits  $31 \sim 28, 8 \sim 4$  of  $aa_{31}$  are matched.
  - iv) Similarly, for the remaining  $(m_0, b_{33}, m_3, b_{25})$ , compute  $aa_{30}$ ,  $aa_{29}$  and check the matching. If all bits are matched and the carried number assumptions are correct, a pseudo-preimage is found.

5) If no pseudo-preimage is found, change the values of the message words in step 2, and repeat steps  $3\sim4$ .

#### 5.2 Complexity Evaluation

One step computation is regarded as  $\frac{1}{48\times 2} = \frac{1}{96}$ RIPEMD compression function computation. The complexity of the attack can be evaluated as follows.

Step 1: Negligible.

Step 2: Negligible.

Step 3(a):  $2^{48} \times \frac{1}{96}$  RIPEMD compression function. Step 3(b):  $2^{48} \times \frac{25}{96}$  RIPEMD compression function. Step 3(c):  $2^{48} \times \frac{1}{96}$  RIPEMD compression function. Step 3(d):  $2^{48} \times \frac{24}{96}$  RIPEMD compression function. Step 3(e):  $2^{48} \times 2 \times \frac{1}{96} + 2^{48} \times 2^3 \times \frac{1}{96} + 2^{48} \times 2^5 \times \frac{1}{96} + 2^{48} \times 2^7 \times \frac{1}{96}$  RIPEMD compression function.

# Step 3(f): Negligible.

Step 4(a):  $2^{40} \times \frac{1}{96}$  RIPEMD compression function. Step 4(b):  $2^{40} \times \frac{1}{96}$  RIPEMD compression function. Step 4(c):  $2^{40} \times \frac{15}{96}$  RIPEMD compression function. Step 4(d) i: Negligible. The number of remaining pairs is  $2^{56} (= 2^{48} \times 2^{40} \times 2^{-32})$ .

Step 4(d) ii:  $2^{56} \times \frac{16}{96}$  RIPEMD compression function. The number of remaining pairs is  $2^{55} (= 2^{56} \times 2^{-8} \times 2^7)$ .

Step 4(d) iii:  $2^{55} \times \frac{1}{96}$  RIPEMD compression function. The number of remaining pairs is  $2^{46} (= 2^{55} \times 2^{-9})$ .

Step 4(d) iv: Approximately  $2^{46}\times \frac{1}{96}$  RIPEMD compression function.

The overall complexity of the above computations is about  $2^{56} \times \frac{16}{96}$ . By performing the attack procedure  $2^{72} (= 2^{128} \times 2^{32} \times 2^{-48} \times 2^{-40})$  times, we expect to get a pseudo-preimage. Therefore the overall complexity of finding a pseudo-preimage of RIPEMD is about  $2^{125.4} (\approx 2^{72} \times 2^{56} \times \frac{16}{96})$  computations. The attack requires  $2^{55}$  ( $m_3$ ,  $b_{25}$ ,  $C^{1\text{st}}$ ,  $aa_{32}$ , D,  $b_0$ ,  $c_0$ ,  $d_0$ )s to be stored, and the memory complexity is  $2^{55} \times 8 = 2^{58}$ words.

The padding rule forces some constraints on the last message words, and there are some restrictions on  $m_{14}$  in our attack. So the padding rule is an obstacle to extend the pseudo-preimage attack to preimage attack on RIPEMD.

#### 6 Conclusions

In this paper, for two double-branch hash functions Extended MD4 and RIPEMD, we examined their security against collision attack and pseudo-preimage attack respectively. A practical attack on Extended MD4 for finding 2-block collision was proposed and a true collision instance of Extended MD4 was found. Based on 8-step initial structure, partial-fixing and partialmatching techniques, etc., a pseudo-preimage attack on RIPEMD was implemented.

A generic method to convert meet-in-the-middle preimage attack to pseudo collision attack was proposed in [55]. On one hand, it greatly improves the number of attacked steps of hash functions, compared with previous collision attack based on differentials and previous meet-in-the-middle preimage attack which needs to take into account padding bits. On the other hand, the method only converts the meet-in-the-middle preimage attack to a pseudo collision attack, not collision attack, and the time complexity is high. Future analysis should be able to explore the security of other double-branch

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hash functions, and to explore other practical relations between preimage attack and collision attack.

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