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Outage Analysis of Opportunistic Cooperative Ad Hoc Networks with Randomly Located Nodes

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Abstract In this paper, an opportunistic cooperative ad hoc sensor network with randomly located nodes is analyzed. The randomness of nodes' locations is captured by a homogeneous Poisson point process. The effect of imperfect interference cancellation is also taken into account in the analysis. Based on the theory of stochastic geometry, outage probability and cooperative gain are derived. It is demonstrated that explicit performance gain can be achieved through cooperation. The analyses are corroborated by extensive simulation results and the analytical results can thus serve as a guideline for wireless sensor network design.

Keywords ad hoc network, selection cooperation, point process

1 Introduction

Wireless sensor networks are widely applied in many areas, such as remote environmental monitoring and target tracking. In most of the applications, sensor nodes are usually randomly distributed. They collect and forward information to their associated server or other nodes. Due to deep fading, sometimes the transmitted data will be impaired and cannot be received correctly. Cooperative communication, which utilizes several other nodes as relays to facilitate the communication between the source and the destination, is an effective technique to combat the severe impairments suffered by radio signals and has been adopted in wireless sensor network to improve communication quality. The study of cooperative communications dates back to 1970s[1-2]. In general, the relay protocols can be classified into decode-and-forward (DF), coded-and-forward (CF) and amplify-and-forward (AF)[3].

Performance analysis for ad hoc sensor networks has been discussed in [4-7]. Most of them consider networks with fixed topology, i.e., sensor nodes' locations are deterministic. However, depending on the applications, the sensor nodes may be randomly deployed. Their locations may also be frequently changed. For example, in target tracking, the sensor nodes are adhered to the objects and will move with the target objects. In these applications, the locations of sensor nodes are irregular and random. This location randomness should be taken into account in network performance analysis. Considering the location randomness of nodes, a mobile ad hoc network is analyzed and outage probability as well as several other network performance metrics are derived in [8]. In [9], an ad hoc network with multi-antenna is analyzed. Adopting cooperation into the ad hoc sensor networks, outage performance and transmission capacity is analyzed in [10].

In this paper, an opportunistic cooperative ad hoc sensor network with randomly located nodes is investigated. Different from [10], the receiver nodes are assumed to possess certain interference cancellation capability. This is reasonable and practical since ad hoc sensor networks are usually interference dominant and interference cancellation for ad hoc networks has been proposed and studied in [9, 11-12]. By modeling the randomly located nodes as a homogeneous Poisson

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point process, the outage probability and cooperative gain are analyzed. The effect of imperfect interference cancellation is also taken into account in the analysis. The correctness of the analysis is verified by simulation results. The analytical results can thus serve as a guidance for wireless sensor network design.

The rest of this paper is organized as follows. The system model is introduced in Section 2. The outage performance of this network is analyzed in Section 3 and the cooperative gain is discussed in Section 4. Finally, conclusions are drawn in Section 5.

2 System Model

In this paper, an opportunistic cooperative ad hoc sensor network is analyzed. The sensors are randomly distributed and thus are modeled as a homogeneous Poisson point process on the plane $\mathcal{R}^2$. Using the popular ALOHA medium access scheme, each sensor is selected as a source node independently with a fixed probability and transmits information to its associated destination with power P. In the considered network as shown in Fig.1, the sensor nodes which have no information to transmit or cannot access the medium are regarded as idle nodes and will facilitate the communications between the source nodes and the destination nodes. Clearly the source nodes can be regarded as an independent thinning of the original homogeneous point process, which results in another homogeneous Poisson point process. This point process formed by the source nodes is denoted as Φ_1 with intensity of λ_1. Similarly, the point process formed by idle nodes is another independent thinning of the original homogeneous Poisson point process and is denoted by Φ_2 with intensity of λ_2. Due to the homogeneity of the sensor nodes, the outage performance of the sensor network can be well characterized by that of a typical link with the source located at the origin and its associated destination located on the horizontal axis as shown in Fig.1. The distance between the source and its destination of the typical link is D.

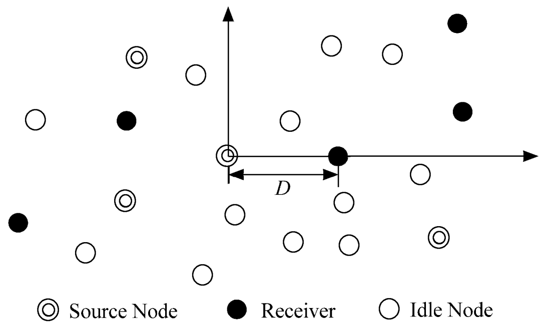

Fig.1. Opportunistic cooperative ad hoc sensor network.

The channel of each communication link involves both large-scale path loss and small-scale fading effects. For an arbitrary link with the source located at s and the destination located at y_d, the channel power gain can be written as $h_{sy_d}\ell(|s-y_d|)$, where h_{sy_d} captures the small-scale rayleigh fading and thus is modeled as an exponential random variable with unit mean, while $\ell(|s-y_d|)=\|s-y_d\|^{-\alpha}$ represents the path loss effect and α is the path loss exponent with $\alpha>2$[13]. Sensor networks are usually interference-dominant and thus noise can be neglected. Then the signal-to-interference-ratio (SIR) seen at the typical destination equals

$$\gamma_{sy_d}=\frac{h_{sy_d}P\ell(|s-y_d|)}{I_{y_d}}, \tag{1}$$

where I_{y_d} is the interference seen at the destination y_d and is given by

$$I_{y_d}=\sum_{s_i\in\Phi_I}h_{s_iy_d}P\ell(|s_i-y_d|). \tag{2}$$

In (2), Φ_I denotes the point process formed by the interferers. In order to improve the communication performance in the interference-dominant network, certain interference cancellation method is adopted in each destination. Similarly to [11], an interference cancellation zone with a radius L is thus formed and its radius is determined by the adopted interference cancellation scheme. Namely, the interference caused by interfering nodes within a distance of L from each destination can be canceled. Usually perfect cancellation is difficult to achieve in practice. To be practical, we assume only certain portion of the interference coming from the zone can be canceled and therefore a factor κ is adopted here to indicate the portion of interference which can be canceled. Clearly, the interference cancellation capability at each destination is characterized by the parameters L and κ, which depends on the cancellation method adopted.

To facilitate the communication between the sources and the destinations, opportunistic cooperation is adopted here and each communication is completed in two phases. In phase one, the source node broadcasts information to its associated destination. The idle nodes which can decode the signal from the source correctly will serve as potential relays of this specific source. In phase two, a node is selected from the source node and its potential relay set as a relay to retransmit the information. Following the widely used best forward channel selection criterion, the node with the best channel condition between it and the destination is chosen to retransmit. With this cooperative scheme, the destination will receive two copies of the original signal and the signal can be recovered by selecting and decoding the one with higher quality, i.e., higher SIR. It is generally assumed in [11, 14-16] that the signal can be correctly decoded only when the received SIR is larger than a threshold T, which is determined by the coding and modulation schemes. Based on the above model, the outage performance of this cooperative sensor network with interference cancellation capability will be analyzed.

3 Outage Analysis

Consider the typical link with the source node located at the origin as shown in Fig.1. Since the information transmission process is completed in two phases, the outage probability corresponding to each phase will be derived first, based on which the outage performance for the typical link can be analyzed.

3.1 Outage Probability of Phase One

In phase one, the source node directly transmits information to its associated destination and the received SIR at the receiver is given by

$$\gamma_{oy_d,1} = \frac{h_{oy_d,1}P\ell(|y_d|)}{I_{y_d,1}}, \tag{3}$$

where y_d is the location of the associated receiver, and $I_{y_d,1}$ is the interference seen at the receiver and can be formulated as

$$\begin{aligned} I_{y_d,1} = & \sum\nolimits_{s_i\in\Phi_{I_{11}}} h_{s_iy_d}P\ell(|s_i-y_d|)+ \\ & \sum\nolimits_{s_i\in\Phi_{I_{12}}} (1-\kappa)h_{s_iy_d}P\ell(|s_i-y_d|). \end{aligned} \tag{4}$$

In (4), $\Phi_{I_{11}}$ is the set of interfering nodes outside the interference cancellation zone and is equal to $\Phi_1 \cap \mathcal{R}^2/\{o\cup b(y_d,L)\}$, where o is the origin and $b(y_d,L)$ denotes a circle with a radius of L and centered at y_d. $\Phi_{I_{12}}$ represents the set of the interfering nodes located in the interference cancellation zone and can be written as $\Phi_{I_{12}} = \Phi_1 \cap b(y_d,L)/\{o\}$. Clearly the second term in (4) is the residual interference after interference cancellation. According to the definition of Poisson point processes, $\Phi_{I_{11}}$ and $\Phi_{I_{12}}$ are two mutually independent Poisson point processes defined in two disjointed areas S_1 and S_2, respectively[17]. The outage probability for phase one can then be expressed as

$$\mathrm{P_{out1}} = \mathrm{P}(\gamma_{oy_d,1} < T) = \mathrm{P}\Big(\frac{h_{oy_d,1}P}{D^\alpha I_{y_d,1}} < T\Big). \tag{5}$$

Since $h_{oy_d,1}$ is an exponential random variable with unit mean, (5) can be rewritten as

$$\begin{aligned} \mathrm{P_{out1}} =& 1-\mathrm{E}[e^{-\frac{TD^\alpha}{P}I_{y_d,1}}] \\ =& 1-\mathrm{E}\Big[e^{-\sum\limits_{s_i\in\Phi_{I_{11}}} h_{s_iy_d}TD^\alpha\ell(|s_i-y_d|)} \times \\ & e^{-\sum\limits_{s_i\in\Phi_{I_{12}}}(1-\kappa)h_{s_iy_d}TD^\alpha\ell(|s_i-y_d|)}\Big] \\ \overset{(a)}{=}& 1-\mathrm{E}\Big[e^{-\sum\limits_{s_i\in\Phi_{I_{11}}} h_{s_iy_d}TD^\alpha\ell(|s_i-y_d|)}\Big]\times \\ & \mathrm{E}\Big[e^{-\sum\limits_{s_i\in\Phi_{I_{12}}}(1-\kappa)h_{s_iy_d}TD^\alpha\ell(|s_i-y_d|)}\Big], \end{aligned} \tag{6}$$

where (a) follows from the independence between $\Phi_{I_{11}}$ and $\Phi_{I_{12}}$. By taking $h_{s_iy_d}$ as a mark for each source located at s_i, we can form two independent marked Poisson point processes based on $\Phi_{I_{11}}$ and $\Phi_{I_{12}}$ as $\hat{\Phi}_{I_{11}} = \{(s_i,h_{s_i})\}$ and $\hat{\Phi}_{I_{12}} = \{(s_i,h_{s_i})\}$ in $S_1\times\mathcal{R}$ and $S_2\times\mathcal{R}$, respectively. For a marked Poisson point process $\hat{\Phi}$ defined on $S\times\mathcal{R}$, its Laplace functional is defined as[16]

$$\mathrm{E}\Big[e^{-\sum\limits_{(s_i,h_{s_iy_d})\in\hat{\Phi}} h_{s_iy_d}f(s_i)}\Big] = e^{-\lambda\int_S (1-\mathrm{E}(e^{-h_{s_iy_d}f(s_i)}))\mathrm{d}s_i}, \tag{7}$$

where λ is the intensity of $\hat{\Phi}$ and $f(s_i)$ is a positive real function of s_i. It directly follows that the second term in the last equation of (6) is the product of the Laplace functionals of $\hat{\Phi}_{I_{11}} = \{(s_i,h_{s_i})\}$ and $\hat{\Phi}_{I_{12}} = \{(s_i,h_{s_i})\}$. With (7) and Slivnyak theorem[16], the outage probability in phase one can be derived as (8).

$$\begin{aligned} \mathrm{P_{out1}} =& 1-e^{-\lambda_1\int_{\mathcal{R}^2/b(y_d,L)}(1-\mathrm{E}_{h_{s_iy_d}}[e^{-h_{s_id}\ell(|s_i-y_d|)TD^\alpha}])\mathrm{d}s_i} \times \\ & e^{-\lambda_1\int_{b(y_d,L)}(1-\mathrm{E}_{h_{s_iy_d}}[e^{-(1-\kappa)h_{s_id}\ell(|s_i-y_d|)TD^\alpha}])\mathrm{d}s_i} \\ =& 1-e^{-\lambda_1\int_0^{2\pi}\int_L^\infty\frac{r}{1+r^\alpha/(TD^\alpha)}\mathrm{d}r\mathrm{d}\theta}\times \\ & e^{-\lambda_1\int_0^{2\pi}\int_0^L\frac{r}{1+r^\alpha/((1-\kappa)TD^\alpha)}\mathrm{d}r\mathrm{d}\theta} \\ =& 1-e^{-\lambda_1 2\pi\int_L^\infty\frac{r}{1+r^\alpha/(TD^\alpha)}\mathrm{d}r}\times \\ & e^{-\lambda_1 2\pi\left(\int_0^\infty\frac{r}{1+r^\alpha/((1-\kappa)TD^\alpha)}\mathrm{d}r-\int_L^\infty\frac{r}{1+r^\alpha/((1-\kappa)TD^\alpha)}\mathrm{d}r\right)}. \end{aligned} \tag{8}$$

According to 3.194 3 and 3.197 2 in [18], the outage probability in phase one can be further simplified in closed-form as (9),

$$\begin{aligned} \mathrm{P_{out1}} =& 1-e^{-\frac{2\pi\lambda_1}{\alpha}TD^\alpha L^{2-\alpha}B(1-\frac{2}{\alpha},1){}_2F_1(1-\frac{2}{\alpha},1;-\frac{2}{\alpha};-\frac{TD^\alpha}{L^\alpha})}e^{-\frac{2\pi\lambda_1}{\alpha}((1-\kappa)TD^\alpha)^{\frac{2}{\alpha}}B(\frac{2}{\alpha},1-\frac{2}{\alpha})}\times \\ & e^{\frac{2\pi\lambda_1}{\alpha}(1-\kappa)TD^\alpha L^{2-\alpha}B(1-\frac{2}{\alpha},1){}_2F_1(1-\frac{2}{\alpha},1;-\frac{2}{\alpha};-\frac{(1-\kappa)TD^\alpha}{L^\alpha})}, \end{aligned} \tag{9}$$

where $B(\rho,\psi)$ is the beta function defined as $\int_0^1 t^{\rho-1}(1-t)^{\psi-1}\mathrm{d}t$, ${}_2F_1(\rho,\psi;\eta;\phi)$ is the Gauss hypergeometric function defined as $\frac{1}{B(\psi,\eta-\psi)}\int_0^1 t^{\psi-1}(1-t)^{\eta-\psi-1}(1-t\phi)^{-\rho}\mathrm{d}t$.

3.2 Outage Probability of Phase Two

The analysis of phase two is more complex than that in phase one, since relay selection is incorporated. To proceed, the distributions of the transmitters/interferers and the potential relays are necessary. They will be discussed first in the following.

3.2.1 Distribution of Potential Relays

In the considered cooperative sensor network, the idle nodes can serve as potential relays for the typical source node only when they can decode the information from the source node correctly. Thus the probability that an idle node located at x serves as a potential relay for the typical source node at the origin can be given as

$$\mathrm{P}_{x,\mathrm{relay}} = \mathrm{P}(\gamma_{ox} \geqslant T) = \mathrm{P}\Big(\frac{h_{ox,1}Pl(|x|)}{I_{x,1}} \geqslant T\Big), \quad (10)$$

where $I_{x,1}$ is the interference seen at x in phase one. With (7), the probability $\mathrm{P}_{x,\mathrm{relay}}$ can be derived as

$$\begin{aligned}\mathrm{P}_{x,\mathrm{relay}} &= \mathrm{E}\Big[e^{-\frac{T\|x\|^\alpha}{P}I_{x,1}}\Big]\\ &= e^{-\lambda_1\int\limits_{\mathcal{R}^2}(1-\mathrm{E}_{h_{s_i y_d}}[e^{-h_{s_i}d^{\ell}(|s_i-y_d|)T\|x\|^\alpha}])\mathrm{d}s_i}\\ &= e^{-\lambda_1 T^{2/\alpha}C(\alpha)||x||^2}, \quad (11)\end{aligned}$$

where $C(\alpha) = \frac{2\pi\Gamma(2/\alpha)\Gamma(1-2/\alpha)}{\alpha}$, $\Gamma(z)$ is the Gamma function defined as $\int_0^\infty e^{-t}t^{z-1}\mathrm{d}t (z>0)$[16]. It implies that the potential relays of the typical source o form a heterogeneous Poisson point process Φ_{or} with mean measure $\Lambda(A) = \lambda_2\int_A \mathrm{P}_{x,\mathrm{relay}}\mathrm{d}x$, where A is the area where the potential relays reside[14-15]. Thus the probability that an idle node located at x belongs to the set Φ_{or} equals

$$\mathrm{P}(x\in\Phi_{or}) = \mathrm{P}_{x,\mathrm{relay}}. \quad (12)$$

3.2.2 Distribution of Transmitters

Since the transmitters in phase two are chosen from the sets of the source nodes and their potential relays, the point process formed by the transmitters in phase two can be regarded as a thinning of a homogeneous point process Φ_3 formed by the superposition of two homogeneous Poisson point processes Φ_1 and Φ_2. As shown in [17], Φ_3 is a homogenous Poisson point process with intensity $\lambda_1+\lambda_2$. Since different transmitters are chosen from different point sets and experience independent channel conditions, the dependence among different transmitters in phase two is weak. Thus we can approximate the thinning for the transmitters in phase two as an independent thinning, and the accuracy of this approximation will be verified by simulations. Moreover, we have the following lemma for the point process Φ_3.

Lemma 1. *The probability that a point of Φ_3 is chosen as a transmitter in phase two is location independent.*

Proof. See Appendix. □

It follows that the thinning of Φ_3 is not only independent but also location independent. According to [17], independent thinning of a homogeneous Poisson point process with a location independent retention probability is still a homogeneous Poisson point process. Thus the point process formed by the transmitters in phase two is a homogeneous Poisson point process. For each source node, there is definitely a corresponding transmitter in phase two, thus the number of transmitters in phase two is equal to the number of the source nodes in phase one. Consequently, the intensity of the homogeneous Poisson point process formed by the transmitters in phase two is equal to λ_1.

3.2.3 Outage Probability in Phase Two

The outage probability in phase two is equal to the probability that the typical source and all its potential relays cannot communicate with the destination successfully. As stated before, the potential relays of a source node form a heterogeneous Poisson point process and the outage probability in phase two thus can be written as

$$\begin{aligned}\mathrm{P}_{\mathrm{out2}} = \sum\nolimits_{n=0}^{\infty}\Big\{&\frac{(\Lambda(\mathcal{R}^2))^n}{n!}e^{-\Lambda(\mathcal{R}^2)}\times\\ &\mathrm{P}(\delta_0=0,\delta_1=0,\ldots,\delta_n=0)\Big\}, \quad (13)\end{aligned}$$

where δ_0 is an indicator corresponding to the typical source and takes the value of 1 when the source can communicate with the destination successfully, otherwise it is 0. Similarly, δ_i, $i=1,2,\ldots,n$, are indicators corresponding to the potential relays. $\Lambda(\mathcal{R}^2)$ in (13) is given by

$$\begin{aligned}\Lambda(\mathcal{R}^2) &= \lambda_2\int_{\mathcal{R}^2}\mathrm{P}_{x,\mathrm{relay}}\mathrm{d}x\\ &= \lambda_2\int_{\mathcal{R}^2}e^{-\lambda_1 T^{2/\alpha}||s-x||^2C(\alpha)}\mathrm{d}x\\ &= 2\pi\lambda_2\int_0^\infty e^{-\lambda_1 T^{2/\alpha}r^2C(\alpha)}r\mathrm{d}r\\ &= \frac{\pi\lambda_2}{\lambda_1 T^{2/\alpha}C(\alpha)}, \quad (14)\end{aligned}$$

where λ_2 is the intensity of the Poisson point process formed by the idle nodes. Due to the independence of the channel gains, δ_0 and δ_i's are independent with each other. Thus (13) can be reformulated as

$$\mathrm{P_{out2}} = \sum_{n=0}^{\infty} \frac{(\Lambda(\mathcal{R}^2))^n}{n!} e^{-\Lambda(\mathcal{R}^2)} \mathrm{P}(\delta_0) \prod_{i=1}^{n} \mathrm{P}(\delta_i), \tag{15}$$

where

$$\mathrm{P}(\delta_0) = \mathrm{P}(h_{oy_d,2} Pl(|y_d|) < T I_{y_d,2}), \tag{16}$$

and

$$I_{y_d,2} = \sum_{s_i \in \Phi_{I_{21}}} h_{s_i y_d} P\ell(|s_i - y_d|) + \sum_{s_i \in \Phi_{I_{22}}} (1-\kappa) h_{s_i y_d} P\ell(|s_i - y_d|). \tag{17}$$

In (17), $\Phi_{I_{21}}$ is the set of interfering nodes outside the interference cancellation zone in phase two and $\Phi_{I_{22}}$ represents the set of interfering nodes located in the interference cancellation zone. Similarly to $I_{y_d,1}$, the second term in (17) is the residual interference after interference cancellation. Since the point process formed by the transmitters/interferers in phase two is approximated as a homogeneous Poisson point process with intensity λ_1, $I_{y_d,2}$ has the same distribution as $I_{y_d,1}$. Then $\mathrm{P}(\delta_0)$ can be derived and has the same expression as $\mathrm{P_{out1}}$ in (9).

On the other hand, $\mathrm{P}(\delta_i)$ is the probability that the i-th potential relay cannot communicate with the destination successfully and can be formulated as

$$\mathrm{P}(\delta_i) = \mathrm{P}(h_{x_i y_d} Pl(|x_i - y_d|) < T I_{y_d,2}), \tag{18}$$

where x_i is the location of the i-th potential relay. It can be rewritten as

$$\mathrm{P}(\delta_i) = \int_{\mathcal{R}^2} \mathrm{P}(h_{xy_d,2} Pl(|x - y_d|) < T I_{y_d,2} | x \in \Phi_{or}) \times \frac{\lambda_2 \mathrm{P}(x \in \Phi_{or})}{\pi \lambda_2 / \lambda_1 T^{2/\alpha} C(\alpha)} \mathrm{d}x. \tag{19}$$

Substituting (11) into (19) and using 3.339 in [18], after tedious computation, $\mathrm{P}(\delta_i)$ can be derived as shown in (20).

$$\mathrm{P}(\delta_i) = 1 - \frac{2e^{-\lambda_1 T^{2/\alpha} C(\alpha) D^2}}{1/\lambda_1 T^{2/\alpha} C(\alpha)} \int_0^{\infty} e^{-\frac{2\pi\lambda_1}{\alpha} T r^{\alpha} L^{2-\alpha} B(1-\frac{2}{\alpha},1)\,{}_2F_1(1-\frac{2}{\alpha},1;-\frac{2}{\alpha};-\frac{Tr^{\alpha}}{L^{\alpha}})} e^{-\frac{2\pi\lambda_1}{\alpha}((1-\kappa)Tr^{\alpha})^{\frac{2}{\alpha}} B(\frac{2}{\alpha},1-\frac{2}{\alpha})} \times e^{\frac{2\pi\lambda_1}{\alpha}(1-\kappa) T r^{\alpha} L^{2-\alpha} B(1-\frac{2}{\alpha},1)\,{}_2F_1(1-\frac{2}{\alpha},1;-\frac{2}{\alpha};-\frac{(1-\kappa)Tr^{\alpha}}{L^{\alpha}})} e^{-\lambda_1 T^{2/\alpha} C(\alpha) r^2} I_0(-2D\lambda_1 T^{2/\alpha} C(\alpha) r) r \mathrm{d}r. \tag{20}$$

Since the potential relays are independent and identically distributed (i.i.d.), δ_i's are i.i.d. random variables. Thus the outage probability in phase two can be derived by substituting (20) into (15). Although (20) is not a closed form, it can be calculated numerically. In the following special case, a closed form for $\mathrm{P}(\delta_i)$ can be obtained.

Special Case. When the destination has no interference cancellation capability, i.e., $L = 0$, the sensor network reduces to a conventional cooperative sensor ad hoc network. Replacing L in (20) as 0, $\mathrm{P}(\delta_i)$ can be derived as

$$\mathrm{P}(\delta_i) = 1 - \frac{2e^{-\lambda_1 T^{2/\alpha} C(\alpha) D^2}}{1/\lambda_1 T^{2/\alpha} C(\alpha)} \int_0^{\infty} e^{-\frac{2\pi\lambda_1}{\alpha} T^{\frac{2}{\alpha}} B(\frac{2}{\alpha},1-\frac{2}{\alpha}) r^2} \times e^{-\lambda_1 T^{2/\alpha} C(\alpha) r^2} I_0(2D\lambda_1 T^{2/\alpha} C(\alpha) r) r \mathrm{d}r. \tag{21}$$

From the property of beta function that

$$\mathrm{B}\Big(\frac{2}{\alpha}, 1-\frac{2}{\alpha}\Big) = \frac{\Gamma(\frac{2}{\alpha})\Gamma(1-\frac{2}{\alpha})}{\Gamma(1)} = \Gamma\Big(\frac{2}{\alpha}\Big)\Gamma\Big(1-\frac{2}{\alpha}\Big), \tag{22}$$

(21) can be reformulated as

$$\begin{aligned}\mathrm{P}(\delta_i) &= 1 - \frac{2e^{-\lambda_1 T^{2/\alpha} C(\alpha) D^2}}{1/\lambda_1 T^{2/\alpha} C(\alpha)} \int_0^{\infty} e^{-2\lambda_1 T^{2/\alpha} C(\alpha) r^2} \times I_0(2D\lambda_1 T^{2/\alpha} C(\alpha) r) r \mathrm{d}r \\ &\overset{(a)}{=} 1 - \frac{e^{-\frac{3}{4}\lambda_1 T^{2/\alpha} C(\alpha) D^2}}{\sqrt{2\lambda_1 T^{2/\alpha} C(\alpha) D^2}} \times M_{-\frac{1}{2},0}\Big(\frac{\lambda_1 T^{2/\alpha} C(\alpha) D^2}{2}\Big),\end{aligned} \tag{23}$$

where (a) follows from 6.6143 in [18], $M_{\lambda,\mu}(\cdot)$ is the Whittaker function with the following property[18]:

$$M_{\lambda,\mu}(z) = z^{\mu+\frac{1}{2}} e^{-\frac{z}{2}} \Phi\Big(\mu - \lambda + \frac{1}{2}, 2\mu + 1; z\Big), \tag{24}$$

and $\Phi(.,.;.)$ is the confluent hypergeometric function defined as

$$\Phi(u, v; z) = \frac{2^{1-v}}{B(u, v-u)} \times \int_{-1}^{1} (1-t)^{v-u-1} \times (1+t)^{u-1} e^{\frac{1}{2}zt} \mathrm{d}t, \tag{25}$$

and has the property

$$\Phi(u, u; z) = e^z. \quad (26)$$

Based on (24) and (26), the expression of $\mathrm{P}(\delta_i)$ can be simplified as

$$\begin{aligned}\mathrm{P}(\delta_i) &= 1 - \frac{e^{-\lambda_1 T^{2/\alpha} C(\alpha) D^2}}{2} \times \\ &\quad \Phi\Big(1, 1; \frac{\lambda_1 T^{2/\alpha} C(\alpha) D^2}{2}\Big) \\ &= 1 - \frac{e^{-\frac{\lambda_1 T^{2/\alpha} C(\alpha) D^2}{2}}}{2}. \quad (27)\end{aligned}$$

Substituting (9) and (27) into (15), the outage probability in phase two can thus be derived as

$$\begin{aligned}\mathrm{P}_{\mathrm{out2}} &= \Big(1 - e^{-\frac{\lambda_1 T^{2/\alpha} C(\alpha) D^2}{2}}\Big) \times \\ &\quad e^{-\Lambda(\mathcal{R}^2) e^{-\frac{\lambda_1 T^{2/\alpha} C(\alpha) D^2}{2}}}. \quad (28)\end{aligned}$$

The analytical results are verified by simulations as shown in Fig.2. In the simulations, the network parameters are set as $\lambda_2 = 0.01$, $L = 3$, $\kappa = 0.9$, $\alpha = 4$, $D = 3$. As shown in Fig.2, the outage probability increases with the intensity of the source nodes, since a larger intensity of source nodes causes a higher interference and thereby a higher outage probability. On the other hand, a larger SIR threshold leads to a higher outage probability as expected. Furthermore, the theoretic results coincide with the simulation results well, which verifies the correctness of our analysis.

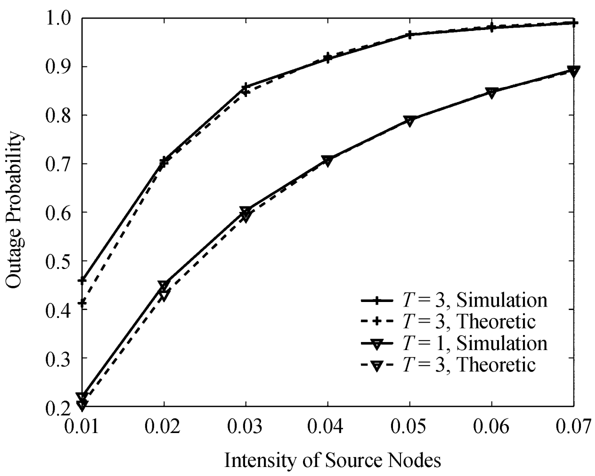

Fig.2. Outage probability in phase two vs the intensity of source nodes.

The relationship between the outage probability in phase two and the intensity of the idle nodes is shown in Fig.3. As expected, the outage probability decreases with the intensity of idle nodes increasing since a larger intensity of idle nodes means more potential relays are available and thus results in a higher spatial diversity. Moreover, Fig.3 shows that a larger transceiver distance also leads to a higher outage probability.

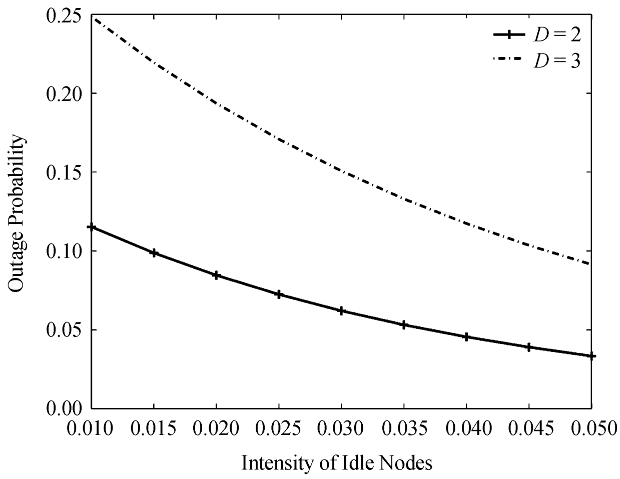

Fig.3. Outage probability in phase two vs the intensity of idle nodes.

Since selected combining is adopted at the receiver, the outage probability of the whole process can be directly computed as[14]:

$$\mathrm{P}_{\mathrm{out}} = \mathrm{P}_{\mathrm{out1}} \mathrm{P}_{\mathrm{out2}}. \quad (29)$$

4 Cooperative Gain

For completeness, the opportunistic cooperative transmission scheme in this paper is compared with the non-cooperative retransmission scheme. In the non-cooperative retransmission scheme, each source node resends the message to its associated receiver by itself rather than by the potential relays in phase two, namely no cooperation is adopted among the nodes. Denoting the outage probability for the non-cooperative retransmission as $\mathrm{P}_{st,\mathrm{out}}$, it can be written as

$$\mathrm{P}_{st,\mathrm{out}} = \mathrm{P}\{\gamma_{s1} \geqslant T, \gamma_{s2} \geqslant T\}, \quad (30)$$

where γ_{s1} is the SIR at the receiver in phase one and γ_{s2} is the SIR at the receiver in phase two.

Due to the independence between the Rayleigh fading in the two phases and according to [14], $\mathrm{P}_{st,\mathrm{out}}$ can be derived as

$$\mathrm{P}_{st,\mathrm{out}} = {\mathrm{P}_{\mathrm{out1}}}^2. \quad (31)$$

For comparison, we define a performance metric, i.e., cooperative gain, as the ratio between the outage probabilities of the non-cooperative and the cooperative schemes as follows:

$$G = \frac{\mathrm{P}_{st,\mathrm{out}}}{\mathrm{P}_{\mathrm{out}}} = \frac{\mathrm{P}_{st,\mathrm{out}}}{\mathrm{P}_{\mathrm{out1}}\mathrm{P}_{\mathrm{out2}}} = e^{\Lambda(\mathcal{R}^2)(1-\mathrm{P}(\delta_i))}. \quad (32)$$

With this performance metric, the cooperative and the non-cooperative schemes are compared as shown in Fig.4. The parameters are set as $T = 3$, $L = 3$, $\kappa = 0.9$, $\alpha = 4$, $D = 3$. As shown in Fig.4, an obvious cooperative gain can be achieved with the cooperative transmission scheme adopted in this paper, when the source density λ_1 is less than 0.05. Moreover, the cooperative gain increases with the intensity of the idle nodes, i.e., λ_2. This is because a higher intensity of idle nodes will result in more potential relays and thereby higher spatial diversity. On the other hand, the cooperative gain decreases when the intensity of the source nodes increases, since each source node gets less potential relays and its associated destination receives more interference.

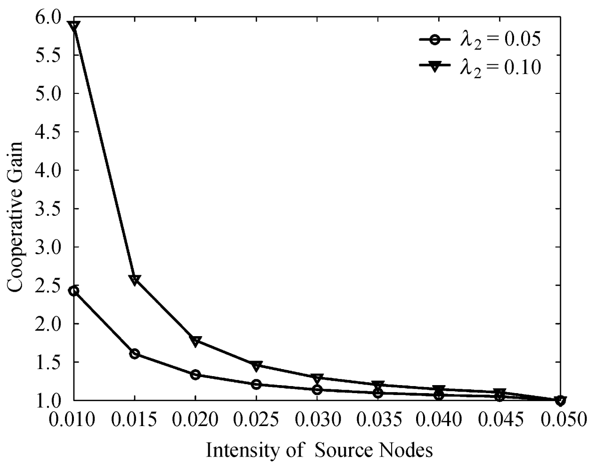

Fig.4. Cooperative gain vs the intensity of source nodes. The parameters are set as $T = 3$, $\alpha = 4$, $D = 3$.

5 Conclusions

In this paper, we analyzed the performance of an opportunistic cooperative ad hoc sensor network with interference cancellation capability. The randomness of nodes' locations and imperfect interference cancellation were taken into account in the analysis. Based on the theory of stochastic geometry, the outage probability was analyzed and the cooperative gain was derived. The correctness of the analytical results has been corroborated by simulation results.

In this work, we only considered the performance of DF protocol with perfect channel state information (CSI). In future, we will try to make an extension to the context with imperfect CSI, which will offer us a more practical way to evaluate the performance of DF protocol.

Appendix Proof of Lemma 1

According to the transmission scheme, given a node located at x, there are two situations for it to be chosen as a transmitter in phase two: 1) the node at x is a source node in phase one and has the best forward channel condition compared with its potential relays; 2) the node at x is a potential relay and has the best forward channel compared with other potential relays as well as its corresponding source node. From the property of Poisson point process, the probability that a point of Φ_1 is located at x equals $\lambda_1 dx$. Similarly, a point of Φ_2 is located at x with probability of $\lambda_2 dx$. Then the probability that a node at x is a source node equals $\frac{\lambda_1}{\lambda_1+\lambda_2}$ and the probability that the node at x is an idle node equals $\frac{\lambda_2}{\lambda_1+\lambda_2}$. It follows that the probability that a node at x can transmit in phase two is

$$\mathrm{P}_x = \frac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{P}_{ts} + \frac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{P}_{ti}, \quad (\mathrm{A1})$$

where P_{ts} is the probability that a source node located at x transmits in phase two, and P_{ti} means the probability that an idle node located at x transmits in phase two. Clearly, the first term in (A1) corresponds to situation 1) and the second term corresponds to situation 2). According to (11) and (12), the probability that an idle node at u serves as a potential relay for the transmitter at x equals

$$\mathrm{P}(u \in \Phi_{xr}) = e^{-\lambda_1 T^{2/\alpha}||u-x||^2 C(\alpha)}.$$

From the property of Poisson point process, P_{st} can be formulated as (A2).

$$\mathrm{P}_{ts} = \sum_{n=0}^{\infty} \frac{e^{-\Lambda(\mathcal{R}^2)}}{n!} \int_{R^2} \prod_{i=1}^{n} \int_{\mathcal{R}^2} \mathrm{P}_{u_i<x}\lambda_2 \times \mathrm{P}(u_i \in \Phi_{xr})\mathrm{d}u_i f(y)\mathrm{d}y, \quad (\mathrm{A2})$$

where n is the number of potential relays of x. By definition, $\mathrm{P}_{u_i<x}$ can be formulated as

$$\mathrm{P}_{u_i<x} = \mathrm{P}\Big(\frac{h_{xy}}{||x-y||^{\alpha}} \geqslant \frac{h_{u_i y}}{||u_i-y||^{\alpha}}\Big) = \frac{||u_i-y||^{\alpha}}{||x-y||^{\alpha}+||u_i-y||^{\alpha}}. \quad (\mathrm{A3})$$

With (11) and (A3), (A2) can be reformulated as (A4),

$$\mathrm{P}_{ts} = \sum_{n=0}^{\infty} \frac{\lambda_2^n e^{-\Lambda(R^2)}}{n!} \times \int_{R^2} \prod_{i=1}^{n} \int_{R^2} \frac{\|u_i-y\|^{\alpha}}{\|x-y\|^{\alpha}+\|u_i-y\|^{\alpha}} \times e^{-K\|x-u_i\|^2}\mathrm{d}u_i f(y)\mathrm{d}y, \quad (\mathrm{A4})$$

where $K = \lambda_1 T^{2/\alpha} C(\alpha)$. Substituting $y' = x - y$ and $u'_i = u_i - x$ into (A4), P_{ts} can be reformulated as (A5).

$$\mathrm{P}_{ts} = \sum_{n=0}^{\infty} \frac{\lambda_2^n e^{-\Lambda(R^2)}}{n!} \times \int_{R^2} \prod_{i=1}^{n} \int_{R^2} \frac{\|u'_i - y'\|^\alpha}{\|y'\|^\alpha + \|u'_i - y'\|^\alpha} \times e^{-K\|u'\|^2} \mathrm{d}u'_i f(y') \mathrm{d}y'. \quad \text{(A5)}$$

In (A5), we have used the fact that $f(y)$ is motion invariant, i.e., $f(x - y') = f(y')$. Clearly the probability P_{ts} in (A5) is independent of the location x. Similar to P_{ts}, P_{ti} can be formulated as shown in (A6).

$$\mathrm{P}_{ti} = \sum_{n=0}^{\infty} \frac{\lambda_2^n e^{-\Lambda(\mathcal{R}^2)}}{n!} \int_{\mathcal{R}^2} \int_{\mathcal{R}^2} \mathrm{P}_{s<x} \mathrm{d}s \times \prod_{i=1}^{n} \int_{\mathcal{R}^2} \mathrm{P}_{u_i<x} \mathrm{P}(u_i \in \Phi_{xr}) \mathrm{d}u_i f(y) \mathrm{d}y, \quad \text{(A6)}$$

where n is the number of potential relays other than the typical one located at x, $\mathrm{P}_{s<x}$ is the probability that x has a better forward channel condition than its corresponding source node at s and equals

$$\mathrm{P}_{s<x} = \mathrm{P}\Big(\frac{h_{xy}}{||x-y||^\alpha} \geqslant \frac{h_{sy}}{||s-y||^\alpha}\Big) = \frac{||s-y||^\alpha}{||x-y||^\alpha + ||s-y||^\alpha}.$$

Following the same derivation for P_{ts}, it is easy to show that P_{ti} is also irrelevant to x when $f(y)$ is motion invariant. Using the result of P_{ts} and P_{ti}, we can conclude from (A1) that the probability P_x is irrelevant to the location of x, which means that the transmitter selection in phase two is location independent.

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