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A Survey on Partial Retrieval of 3D Shapes

Zhen-Bao Liu[1] (刘贞报), Shu-Hui Bu[1,*] (布树辉), Kun Zhou[2] (周　昆), Shu-Ming Gao[2] (高曙明)
Jun-Wei Han[3] (韩军伟), and Jun Wu[4] (吴　俊)

[1] *School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, China*

[2] *College of Computer Science and Technology, Zhejiang University, Hangzhou 310058, China*

[3] *School of Automation, Northwestern Polytechnical University, Xi'an 710072, China*

[4] *School of Electronics and Information, Northwestern Polytechnical University, Xi'an 710072, China*

E-mail: {liuzhenbao, bushuhui}@nwpu.edu.cn; kunzhou@acm.org; smgao@cad.zju.edu.cn; {jhan, junwu}@nwpu.edu.cn

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Abstract Content-based shape retrieval techniques can facilitate 3D model resource reuse, 3D model modeling, object recognition, and 3D content classification. Recently more and more researchers have attempted to solve the problems of partial retrieval in the domain of computer graphics, vision, CAD, and multimedia. Unfortunately, in the literature, there is little comprehensive discussion on the state-of-the-art methods of partial shape retrieval. In this article we focus on reviewing the partial shape retrieval methods over the last decade, and help novices to grasp latest developments in this field. We first give the definition of partial retrieval and discuss its desirable capabilities. Secondly, we classify the existing methods on partial shape retrieval into three classes by several criteria, describe the main ideas and techniques for each class, and detailedly compare their advantages and limits. We also present several relevant 3D datasets and corresponding evaluation metrics, which are necessary for evaluating partial retrieval performance. Finally, we discuss possible research directions to address partial shape retrieval.

Keywords 3D shape, partial retrieval, survey, classification, evaluation

1 Introduction

The speedy update in graphics hardware and 3D model tools, and the popularity of low cost 3D scanners make it easy to acquire, create, and manipulate 3D models. Users can experience a large number of 3D applications including computer aided design, multimedia, game, virtual reality, and films, which leads to the availability of public 3D models on Internet. How to reuse these existent models is a challenging issue. Shape retrieval of 3D shapes has a research history of over ten years, and is still a hot topic in computer graphics, computer aided design, multimedia, and computer vision. Thanks to many proposals from researchers, various global methods have appeared and achieved high retrieval performances on several 3D shape benchmarks such as Princeton Shape Benchmark[1], McGill 3D Shape database[2], NTU 3D Model database[3], Engineering Shape Benchmark[4], Konstanz 3D model database[5], and SHREC datasets. Different from global shape retrieval, the objective of partial retrieval is to compare and search 3D models with similar parts as query. This is fundamentally distinct from global shape retrieval. Fig.1 illustrates the difference between global matching and partial matching. Global matching considers global similarity of shapes (left cup and center cup), while partial matching considers part similarity (handles of left cup and right

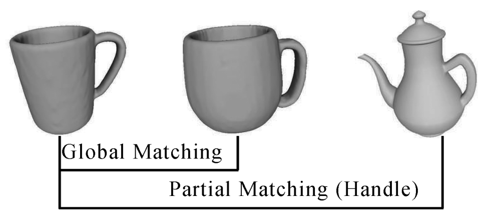

Fig.1. Illustration of the difference between global matching and partial matching.

Survey

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*Corresponding Author

pot) and ignores the global similarity. The problem is very challenging because most 3D objects are not partitioned into meaningful parts and tagged with part labels in advance.

Previous surveys address global 3D shape retrieval[1,5-11], shape recognition[12], shape correspondences[13], non-rigid shape analysis[14], shape segmentation[15-16], shape deformation[17], and symmetry in 3D geometry[18]. And a very recent comprehensive study[19] on rigid and non-rigid registration includes related point features, saliency, and correspondence. These studies excellently organize a large number of state-of-the-art researches and demonstrate the recent developments in shape analysis. However, to the best of our knowledge, there is a lack of a comprehensive discussion on recent advances in partial shape retrieval, especially related with content-based shape retrieval. Although partial shape retrieval has close relationship with correspondence[13], their targets and means are different. Correspondence focuses on how to establish a meaningful mapping between elements of a pair of shapes, while the objective of partial shape retrieval is to efficiently search partially similar shapes with query in a database. Although building local relationships between two shapes is a common characteristic between correspondence and some retrieval methods, correspondence is not concerned to efficiently compare, organize, and index these local features, and also evaluate similarity between two objects. Moreover, in fact many partial retrieval methods do not depend on such local mapping.

Many successful solutions to partial retrieval have appeared recently and absorbed much attention in not only research domain but also industries. In this paper, we review recent work in 3D partial shape retrieval, with a focus on content-based partial retrieval. We leave out global shape retrieval and text caption based part retrieval from our discussion. All the state-of-the-art methods are classified into three classes according to part definition, extracted features, part correspondences, and measures of partial similarity. We discuss the advantages and limitations of each class, and present several public datasets and evaluation metrics for partial retrieval. We hope that our work will not only help a novice to enter this field quickly, but also assist in future choices of partial retrieval algorithms in various applications.

2 Definition of Partial Shape Retrieval

The idea of partial shape retrieval is motivated by text retrieval that searches relevant documents by keywords or sentences. Keywords are seen as parts of the whole document. Likewise, the problem also exists in image and video matching and retrieval. The goal is to detect a moving or static object of interest in a scene where the object might be occluded. The object is a part of the scene, and part retrieval is carried out by detecting its features and segmenting it from the background.

Now we first clarify the terms "partial" and "partial retrieval" in 3D shape domain. The meaning of "partial" is based on the assumption that any object is composed of several semantic parts, and each part can be considered as a "partial" shape. In the occasion of 3D discrete models, a whole shape is represented by a set of elements including vertices, faces, and edges, and "partial" can be represented by a subset of these elements. Let S denote a full set, and s is a subset of it ($s \subset S$). Here we need to emphasize that there is no absolute "partial" shape. For example, a hand can be a full shape, and can also be seen as a part of human shape. If the hand model is considered as a whole shape, then fingers are the subparts of the model. It brings a difficult problem whether the hand should be treated as a partial shape in the stage of partial retrieval. We think this is closely related with specific application context, which commonly contains two tasks, part-in-whole retrieval and whole-to-whole retrieval by partial similarity. Here we will define these two tasks which retrieval methods should finish.

2.1 Part-in-Whole Retrieval

The task of part-in-whole retrieval is to determine whether an input shape is inside a whole shape. The input shape is considered as a subpart cut from a whole shape, or a range scan of a whole shape. The subpart commonly has semantic meanings. A simple example of part-in-whole retrieval is shown in Fig.2(a). We select a tire model as input, and search a car with the similar

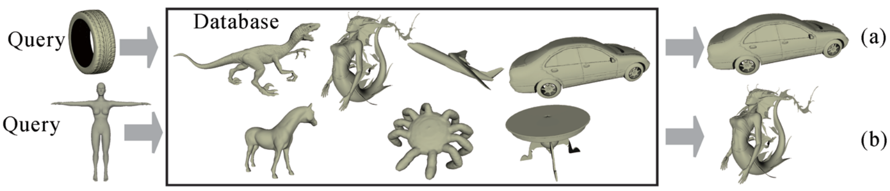

Fig.2. Examples of (a) part-in-whole retrieval and (b) whole-to-whole retrieval by partial similarity. The task of (a) is to match and search a tire to a car in a database. The task of (b) is to match and search a woman to a mermaid in the database.

part in a dataset. In order to define it more accurately, we describe the part-in-whole retrieval in terms of mathematical expressions.

Assume that a 3D partial shape contains an element set s of (V, F, E) in which V denotes vertices, F are faces and E are edges. Let a 3D whole shape be expressed in the form of a set T of (V, F, E) elements, where T contains k parts $\{t_i\}$, $i \in [0, k-1]$. Given s, search T by part-in-whole retrieval. In essence, we need to find t_i, the target part similar to the query s. Therefore, the objective of the related algorithms is to define a dissimilarity measure $D(s, T)$ between partial shape s and complete shape T. If T is explicitly segmented into k parts, the dissimilarity measure $D(s, T)$ can be easily obtained by virtue of comparing the distances $\{d(s, t_i)\}$ between s and each part t_i. When parts of T are not specified, computation of $D(s, T)$ is expressed as a problem of searching an optimal subset with a minimum distance to s in the whole set T.

2.2 Whole-to-Whole Retrieval by Partial Similarity

Different from part-in-whole retrieval, whole-to-whole retrieval by partial similarity is to solve the similarity problem of two global shapes. The aim is to measure partial similarity between two global shapes. It differs from traditional global shape retrieval mentioned in [1, 5-11], because the precondition of traditional global retrieval is that two shapes should have semantic affinity whereas whole-to-whole retrieval based on partial similarity does not care whether two shapes are globally similar. Two shapes in a different semantic category may also be partially similar, that is, we can say they are similar if a subpart of one shape has the same semantics as that of another shape. For example, as illustrated in Fig.2(b), whole-to-whole retrieval by partial similarity can match and search a woman and a mermaid, while traditional global retrieval aims at finding two similar females. Here we give a mathematical definition for the whole-to-whole retrieval by partial similarity.

Let S and T be two 3D overall shapes. We first suppose the structures of two shapes are known. S contains m parts $\{s_i\}, i \in [0, m-1]$, and each part s_i is expressed with a subset of elements such as faces, edges, and vertices. T contains n parts $\{t_j\}$, $j \in [0, n-1]$. We match S and T by partial similarity. Clearly, if two overall shapes are partially similar, there exists at least one pair of similar parts between S and T. The procedure of the task commonly includes: 1) finding one or more optimum corresponding pairs of s_i and t_j, 2) defining a distance measure $d(s_i, t_j)$ between s_i and t_j by global similarity, 3) evaluating the distance $D(S, T)$ between two shapes by partial similarity. While if parts of S and T are not provided explicitly, the problem of whole-to-whole retrieval can be converted to a problem of finding an optimal intersection between two sets S and T.

2.3 Desirable Capabilities

Partial shape retrieval faces many challenges from a variety of 3D models, different organizations of parts, local semantic variation, and so on. Here we highlight the desirable properties of 3D partial shape retrieval.

1) Wide adaptation to shape types like manifold and non-manifold, orientable and non-orientable, closed and open models, and handle multi-resolution models. It also should be robust against small local changes, noise, and topological degeneracy.

2) Invariance with respect to rigid transformation and scale of objects. When objects are rotated, translated, or scaled, ideal algorithms can still recognize their parts.

3) Invariance with respect to non-rigid deformation of objects. The difficult problem is that, when a part is deformed, for example, by isometric transformation, the novel part should be partially matched to the whole shape containing the original part.

4) Local geometric feature extraction with high discriminative power and reliable correspondence between similar regions of different shapes. The feature should efficiently and compactly represent local region, and discriminate different types of local regions.

5) Efficient local region definition and data structure for organization of local regions. Algorithms should provide meaningful definition of local regions, which can be sparse or dense and suitable for fast and accurate partial retrieval. The structure should also help speedy partial search, and have low computational complexity avoiding combinatorial explosion of multi-features matching.

3 Categorization of Partial Shape Retrieval Methods

Compared with global retrieval, partial shape retrieval must address more technical difficulties such as "partial" shape recognition, part correspondences, part organization, and partial similarity measure. There have been many successful solutions in the domains of computer graphics, CAD, computer vision, multimedia, and pattern recognition up to now. We review recent literature and classify the state-of-the-art techniques into three categories: methods based on local descriptors, methods based on segmentation, and methods based on view, according to how to define parts, correspond with parts, extract part features, organize part features, and measure partial similarity.

Methods Based on Local Descriptors. The class of methods solves the problem of partial retrieval by introducing local descriptors or point descriptors such as curvatures, which capture the information of small neighborhood area. They determine partial correspondences by finding a common subset between two sets of different local descriptors, and a partial similarity metric is adopted to compute a match cost between two sets of local descriptors. This type of methods are adaptable to topologically complex objects, and a query patch can be arbitrarily selected and defined even without any meaningful information. It means that they do not require advance segmentation on 3D model.

Methods Based on Segmentation. The class of methods first explicitly define parts by segmenting a 3D shape into meaningful subparts or salient patches, and then compute each subpart signal and organize the structural relations of subparts. Partial similarity is commonly measured by virtue of structural algorithms such as subgraph isomorphism. These methods based on semantic segmentation are effective for searching natural parts with simple topology, and the meaningful matches are consistent with human perception.

Methods Based on View. View-based methods commonly generate a set of 2D images of a 3D model from uniformly distributed viewpoints. Partial shape retrieval is implemented by only comparing views, for example, matching a range image generated by laser scanning to a complete shape. Approaches based on views have the capabilities of supporting 2D image query, 2D sketch query, and range scan query for 3D partial shape retrieval.

4 Local Descriptors Based Partial Shape Retrieval

In this section, we survey the recently proposed local descriptors, with an emphasis on their use for partial shape retrieval. Like 2D local descriptors which play an important role in image and video domain[20], 3D local descriptors are also widely considered successful in shape registration[21], saliency description[22-23], symmetry detection, surface correspondence[13], matching shape in multi-object scenes, and also partial shape retrieval[24]. A local descriptor is commonly a scalar or vector valued function computed for each point on the 3D mesh, for example, Gaussian or mean curvatures and their variants[25-26]. Different from global descriptors representing a whole 3D shape, a local descriptor is only based on a small neighbor region of a point, which means that it is more sensitive to local changes and hence difficult to be extracted stably. Desired point-wise local descriptors are distinctive, robust to rigid transformation and non-rigid deformation, and insusceptible to incomplete objects, which is important for partial retrieval. They do not require segmenting 3D shapes to provide partial cues for partial retrieval.

For those local descriptors relying on the computation of local region, how to define "local" regions of a descriptor is an important issue. Generally, there are several different definitions, one of which is Euclidean spatial neighborhood area, and other alternatives include geodesic ring neighborhood, points in local view, and k-nearest neighbors in unstructured point cloud[27]. Euclidean spatial neighborhood is determined by a set of vertices lying in a ball centering at the analyzed point[21,28-30]. As Euclidean distances are easily computed the determination of neighborhood is fast. In order to acquire local region lying on the surface, one or more rings of neighborhood in a fixed or scaled geodesic distance are commonly used to provide local support for the analyzed point[31-33].

Another key question that needs to be answered is how to define "descriptor" of points on the surface. Descriptor of a point describes its feature and distinctive information of its local region. It is desirable that the regions with visual saliency such as protrusion, tip, and transition between different parts, have larger description values, and flat or smooth regions have smaller values. Coding these regions by saliency is the task of local descriptors. There have been some early work like splash feature, shape index, and point signatures. Next we will investigate recently proposed representative descriptors.

4.1 Local Descriptors

Spin Images. Spin images[34] are generated at all vertices of a model based on an oriented point basis (p, n). The idea is illustrated in Fig.3, where the two coordinates of the basis are α and β. Here α is the perpendicular distance to the normal line L, and β is the signed perpendicular distance to the tangent plane P. The neighborhood vertices are projected onto the

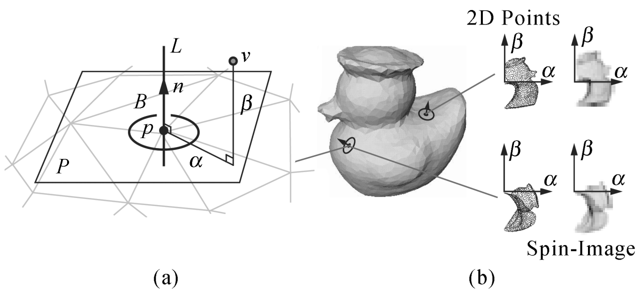

Fig.3. Spin images and its local coordinate system. (a) Local coordinate system definition of spin images[34]. (b) 3D model with 2 sampled spin images.

cylindrical coordinate system, and a 2D image is generated which represents a density histogram of points on the model. For making the descriptor robust to isometric transformations, 3D shape is embedded as a manifold in the specific dimensional intrinsic space by preserving pairwise geodesic distances in the original 3D space, and constructed spin images in the intrinsic space[35]. Spin images have been proved to be suitable for missing surface retrieval[36].

Snapshot Descriptor. The descriptor is generated by taking snapshots of the surface over each point using a virtual camera oriented perpendicularly to the surface around the point[37]. A local coordinate frame are defined at each point on the surface, and three axes of the coordinate frame are based on three eigenvectors of a vertex scatter matrix. Fig.4 demonstrates generation (Figs. 4(a)∼4(b)), pairwise matching (Fig.4(c)), and correspondence result (Fig.4(d)) of snapshots on 3D face models.

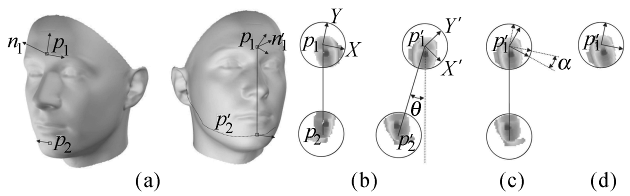

Fig.4. Local snapshot descriptor and matching scheme proposed in [37].

Local Spherical Harmonics Descriptor. Global spherical harmonics descriptor of 3D shape was first proposed by Kazhdan *et al.*[38], which is constructed on several concentric spheres centering at the mass center. Similarly, for local analysis, local spheres with different radii centered in the analyzed point are first generated, and they intersect with mesh surface. A binary identified function is used to evaluate intersection and non-intersection relationship between each sphere and mesh surface. The spherical function is decomposed into harmonic functions with different frequency vectors, which are used to characterize the local surface of the analyzed point[25,28,39]. The descriptor is robust to rotation and translation transformation, nevertheless, it is susceptible to non-rigid deformation.

Laplace-Beltrami Descriptor. To encode enough local context while keeping it isometry-invariant, surface features are extracted based on the eigendecomposition of the Laplace-Beltrami operator defined on manifold[40-43]. For each sampling vertex, the method first defines its neighbors to form a local region. A fixed-area-ratio search strategy is utilized in [41] to prevent local scales. The set of larger eigenvalues for the local region are adopted to form a local descriptor for partial retrieval. This descriptor is isometry-invariant, and consequently it is able to handle non-rigid transformation, and suitable for partial retrieval among non-rigid 3D shapes.

Heat Kernel Descriptor. Recently, the heat kernel descriptor has received increasing interest in the use of diffusion geometry for shape representation. It is derived from a heat diffusion equation by using Laplace-Beltrami operator on surfaces. The fundamental solution of the heat equation, called the heat kernel[44], is used to detect local nonrigid features of surface[45-47] and volumetric representation[48-49]. It is adopted to design an efficient pose oblivious retrieval algorithm for partial and incomplete models[50]. It results in effective partial retrieval, e.g., between human and its arm, or a horse and its leg. Illustration of its effect is shown in Fig.5. Heat kernel is intrinsic and insensitive to isometric deformation, and robust to different genuses, e.g., a cup with one handle and more handles. It captures a local region at an easily controlled scale in the neighborhood of a point on the mesh, and only needs the computation of larger eigenvalues of Laplace-Beltrami operator and their eigenfunctions, which can be calculated with high efficiency. One disadvantage is that it is unable to handle different scales of a shape, and thus scale normalization[51-52] is necessary.

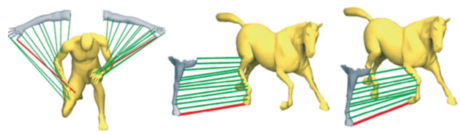

Fig.5. Partial matching using heat kernel local descriptor suggested in [47].

3D Extension from Classical 2D Descriptors. A natural way is to extend 2D image descriptors to 3D mesh structure. For example, several representative 2D descriptors[20] such as 2D Harris descriptor, Difference of Gaussians (DOG), Histogram of Oriented Gradients (HOG), 2D SURF, maximally stable extremal component (MSER), shape context, and SIFT, have been extend to capture 3D local geometric features. A few representative examples are 3D Harris[30], Mesh-DoG and HoG[53-54], geodesic scale space DoG[55], 3D SURF[56], Shape MSER[57], and 3D shape context[58-60], and 3D SIFT[61-62].

Combination and Learning of 3D Descriptors, and Other Descriptors. Combination of different types of local descriptors can improve distinctiveness of a single descriptor, for example, geometric information and photometric information[63], global features and local features[64], visual and geometric features[65-66]. Nevertheless, how to weight different descriptors is a difficult problem. Supervised methods such as Hidden Markov models[31,53] have been applied, which enable learning from training samples to obtain optimum capability of partial retrieval. Other recent descriptors used to search partial information include local rotational symmetry[67], and so on.

Discussion. We theoretically analyze the performances of seven representative local descriptors based on their adopted neighborhood, embedding and mapping space, and analysis tools. The importance is placed on their abilities of dealing with rotation, scale, non-rigid deformation and topological or geometrical noises. We list the sensitivities of these local descriptors in Table 1. They are all rotation invariant and the analyzed shape requires uniformly scaling before feature extraction. According to the difference between embedding spaces like Euclidean space and spectral space, some descriptors are non-rigid and others only handle rigid transformation. Heat kernel signature has another significant characteristic that it is robust to shapes with different genuses[47]. Local descriptors also need to treat noises such as topological noise and geometrical noise. Spin images and snapshots are based on projections from 3D surface to 2D image planes, and small holes on the surface are converted to black pixels in images, which seldom influence traditional 2D descriptions such as Fourier transform while analyzing these images with noisy pixels in the plane domain. The Laplace-Beltrami operator, 3D Harris, and 3D SURF are severely dependent on local connectivity, and holes or gaps may cause large fluctuations of their values. Heat kernel signature is robust to local noise, because it is built on heat diffusion and represents global structure of 3D shape when the time becomes long.

Table 1. Performances of Seven Representative Local Descriptors

Methods	Rotation Invariant	Need Scaling	Non-Rigid Deformation	Noise
Spin image	Yes	Yes	Sensitive	Robust
Snapshot	Yes	Yes	Sensitive	Robust
Local spherical harmonics	Yes	Yes	Sensitive	Robust
Laplace-Beltrami operator	Yes	Yes	Robust	Sensitive
Heat kernel signature	Yes	Yes	Robust	Robust
3D Harris[10]	Yes	Yes	Sensitive	Sensitive
3D SURF[56]	Yes	Yes	Sensitive	Sensitive

4.2 Partial Similarity Measure of Local Descriptors

Partial retrieval is realized by computing the dissimilarity between the input set of local descriptors and the target set of local descriptors[68]. A direct way is to compare all pairs of points of input shape and target shape, and the similarity degree is computed by summing up the distances of each nearest pair. The strategy of pairwise comparison is adopted in partial retrieval methods such as [34, 37], which are considered robust against local shape variations; however, the optimal solution is searched by a brute force approach and matching operation grows exponentially with the feature number increasing.

Sparse Comparison. If only comparing sparse salient points and omitting a large number of points locating on the flat areas, the computation of partial similarity measure can be sped up. Therefore, many algorithms are developed to detect sparse salient points before shape retrieval such as computing extrema of geodesic scale space on the surface[55], sorting representative points by DCG (Discounted Cumulative Gain) values[28] shown in Fig.6, and computing local extrema of Gaussian-weighted average of mean curvatures[69].

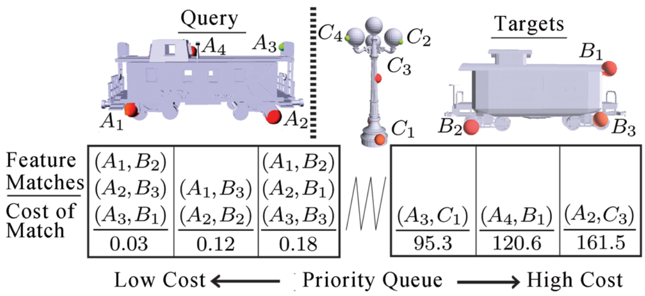

Fig.6. Single priority queue to store partial matches and detect salient points for sparse comparison proposed by Funkhouser *et al.*[79].

Multi-Layer Comparison. A local descriptor is set to multi-layer vector function, where each layer is defined as a neighborhood centering at the analyzed point within a specific distance to the point. The first layer corresponding to the smallest region is firstly compared to compute a dissimilarity distance between the query and the target shape. Then the second layer with a larger region will be searched in the target shape. The algorithm continues until the distance of this layer is above a threshold. This multi-layer descriptor comparison efficiently reduces search and comparison space[25].

Iterative Closest Point. Iterative Closest Point (ICP) is an algorithm commonly used for finding optimal alignment of surfaces[70], which can also be applied to partial retrieval determining whether two rigid or nonrigid shapes are close[71-72]. Two-step ICP based partial retrieval is performed by rejecting or downweighting points with bad correspondence. Although it allows finding the best matching parts, the size of regions for partial retrieval is not explicitly controllable, and also the matching parts are irregular, which can be improved by introducing Hausdorff distance as metric for partial match[73].

Descriptor Clustering and Vector Quantization. In order to resolve the problem of computational cost in the pairwise comparison, it is a natural choice to cluster similar local descriptors before retrieval with unsupervised clustering such as K-means. According to the clustering results of descriptor vectors, vector quantization represents each vector by the index of the cluster that it belongs to. It can be used to reduce comparison times of local descriptors[74].

Bag of Features. Recently, bag of features (BoF) has been borrowed from text and image processing to address correspondence and retrieval of 3D local descriptors[39,43,45,75-78]. Local descriptors are clustered over a large set of 3D models, and each cluster is considered as a codeword. One common method is K-means clustering. Partial similarity of shapes can be obtained by comparing the codeword histograms. Usually, one 3D shape contains self-similar geometric information and a large number of flat points, and thus the 3D shape can be compactly encoded into a vector of occurrence frequencies of visual words, each of which represents a class of similar features. Instead of establishing correspondences for every pair of points, the comparison between bag of features is usually carried out by finding weighted correlation between vectors. Such a method has a fast processing speed and is suitable for indexing and searching 3D parts in a very large database.

Discussion. From the above mentioned descriptors, we can see the task of partial retrieval is to not only propose a compact and discriminative local descriptor, but also define a consistent similarity metric on two different sets of local descriptors. Although retrieval via local descriptors is not limited by partial shape with or without practical semantics, this leads to a problem of high time complexity in finding optimal correspondences although these local descriptors are able to provide detailed geometrical description. The problem can be avoided by adopting some efficient techniques mentioned above, especially in a huge shape database. In addition, if commonness of most local descriptors and connection relationship among neighborhood descriptors could be considered, algorithms are able to avoid many-to-many comparison between two sets of local descriptors, and also increase the discrimination ability on two different sets.

5 Segmentation-Based Partial Shape Retrieval

Distinct from local descriptors based methods, explicitly defining subparts is also a successful solution to partial shape retrieval. Subparts are regarded as components containing rich and compact semantic information. A middle level representation of shape can facilitate the capture of the semantics implied in the shape. It can also filter out the influence of local change of shape. Many researchers have attempted to segment a whole model into different meaningful components in automatic and semi-automatic ways. Partial shape retrieval techniques employ segmentation algorithms such as hierarchical mesh decomposition, shape diameter function, medial surface, Reeb graph, skeleton, and spectral clustering, to provide the definition of subparts[81]. After segmentation, the topological relations of these subparts are stored in a defined structure such as graph in most methods, while there also exist other methods without considering the connections between subparts. Geometric attributes are commonly attached to each subpart, and the attributes are represented by excellent global shape signatures. Matching a subpart in a whole model is finally realized by measuring part similarity. We intend to describe this line of work using three main steps: part definition, part signature, and part organization as follows.

5.1 Part Definition

Defining meaningful or salient parts has a close relationship with shape segmentation. Generally speaking, the way of automatic segmentation determines how to define the subparts of one 3D model for partial retrieval. Many successful segmentation algorithms have been applied into following partial retrieval techniques, for example, octant-based uniform segmentation in [82], greedy clustering based on geometric criterion used in [83-86], protrusion-oriented segmentation[87], part decomposition via shape diameter function[80] (see Fig.7), part extraction in medial surface[88], partial shape retrieval method that works with skeletal representations[89] (see Fig.8), and shape subpart definition by extracting Reeb graph[90-94] (see Figs. 9(a)$\sim$9(d)). Although part definition via these methods provides efficient and compact structures which simplify partial search, there are also several limitation factors which may affect the performance of partial retrieval, which are listed in Table 2.

5.2 Part Signature

Geometric features computed from the shape of the parts are commonly used to characterize each part segmented in the previous step, and obtain compact feature description. The category of partial retrieval methods employs many sorts of shape geometric characteristics in order to extract the signature of each part. These geometric characteristics have an important

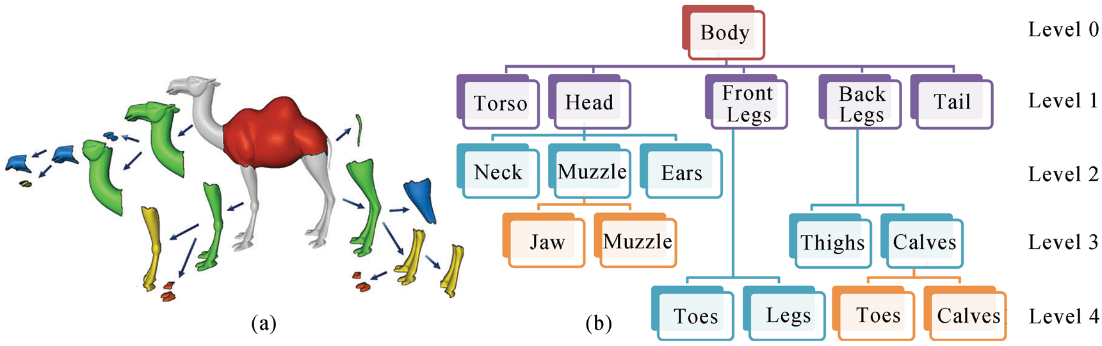

Fig.7. Hierarchical graph of parts segmented by shape diameter function in [80].

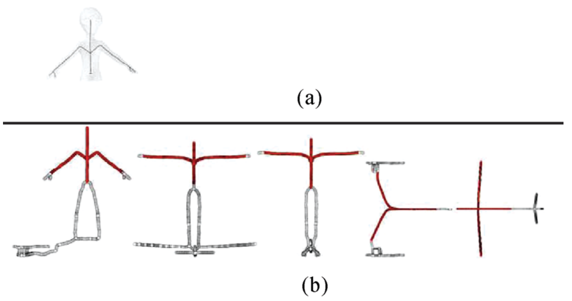

Fig.8. Partial matching using curve skeletons proposed by Cornea *et al.*[89]

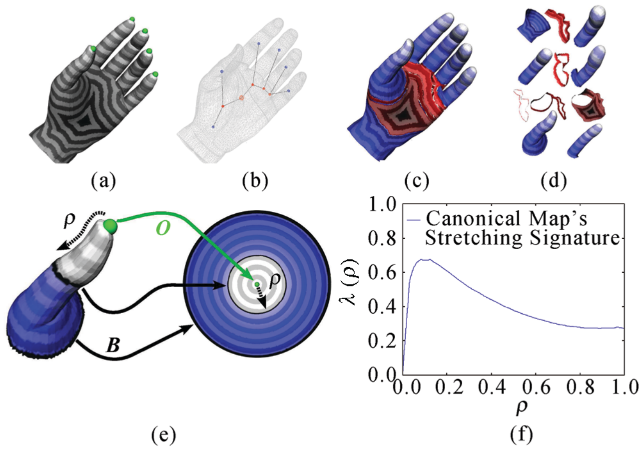

Fig.9. Partial matching using Reeb graph unfolding taken from [91].

Table 2. Limitation Factors Affecting Partial Retrieval in the Step of Part Definition

Part Definition	Limitation
Greedy clustering	Difficult to set desired number of parts
Shape diameter function	Unable to define parts of many open models and CAD models
Medial surface	Extra voxelization and intrinsic sensitivity to small boundary changes
Curve skeleton	Extra voxelization and unstability in preserving topology
Reeb graph	Sensitive to threshold settings

effect on distinguishing dissimilar parts, and discriminative characteristics will promote the accuracy of retrieval parts. Tangelder and Veltkamp[6] surveyed many global descriptors in detail. Shilane *et al.*[1] compared 12 early representative algorithms with respect to processing time, storage requirements, and discriminative power on Princeton Shape Benchmark. The global signatures relevant to partial retrieval methods include distance distribution (e.g., Euclidean distance[95], geodesic distance[96], and biharmonic distance[97]) used in [80, 84], shape diameter function adopted to recognize parts[80], conformal geometry signature identifying convex parts[80], Reeb chart unfolding signature characterizing each segmented Reeb chart[91] (see Figs. 9(e)~9(f)), spherical harmonics descriptor[38,98] adopted to generate part signature[85,92], super-ellipsoid fitting parameters[88], and other variants[83] of basic surface feature like curvature.

5.3 Part Organization

In order to efficiently search a particular part in a whole shape, researchers have developed several ways to organize parts, such as graph, hash table, thesaurus and vocabulary, to capture the topological relationship between parts. These methods are usually represented through graph data structures. Thus, the task of finding similar parts is simplified to solve the problem of subgraph matching or subgraph isomorphism. Each node of the graph is associated with part signature previously mentioned. Alternatively, parts can be organized in a hash table, thesaurus and vocabulary, which can speed up matching and retrieval of parts. We next describe two representative types of part organization.

Subgraph Isomorphism. Assume that a 3D shape is represented by virtue of a topologically connected graph consisting of nodes and edges, such as common undirected graph adopted in [88], Reeb graph used in [91-92], bipartite graph used in [80, 99], skeleton graph adopted in [89], binary tree used in [100], feature dependency directed acyclic graph[101], attributed relational graph[87,102], and hierarchical assembly structure[103]. Each segmented meaningful part is identified by a single node, and edges in the graph represent adjacency relations between these segments. Therefore, the problem of partial retrieval of 3D shapes is easily solved by virtue of detecting subgraph isomorphism and matching subgraphs of two shapes. Searching a partial shape in a whole shape is realized by determining whether a smaller graph is identical to any of subgraphs of a larger graph, which is also considered as a problem of Maximum Common Subgraph[104]. Nevertheless, subgraph isomorphism requires attaching a geometric attribute to each node for 3D partial retrieval. At the same time, graph matching should accommodate intrinsic variability of 3D shapes under consideration, and noise resulting from the graph extraction process. It should be also noted that many mechanical models in CAD domain are designed by feature-based 3D modelers, and parts and their relationship in each model are represented with feature types (e.g., concave, blend, passage, chamfer and smooth-out in [105]) and their feature graphs.

As a result, in order to measure the similarity of two graphs, inexact matching methods such as spectral methods, tree search methods used in [91-92], and optimization methods used in [88-89], are introduced to reduce the computational cost in exact graph match and efficiently match parts of 3D models. Fig.8 illustrates the process of partial retrieval proposed by Cornea *et al.*[89] using part curve skeleton of Fig.8(a) to search its correspondence parts Fig.8(b) from a skeleton database. The graph representation in [88] is shown in Fig.10. The authors adopted a subgraph matching technique proposed in [106], aiming to cast discrete optimization of subgraph matching into a continuous optimization problem. The reason of doing this is that many existent optimization methods could be used to find an optimal or suboptimal solution. One drawback of the methods adopting common subgraph isomorphism is the sensitivity to slight topological differences with the same semantics.

Hash Table, Thesaurus and Vocabulary. Partial features were previously stored in a retrieval database without organization[107]. Recently, efficient data structures like hash table, thesaurus, and vocabulary, are also introduced to organize the set of part signatures. An efficient data structure, geometric hash table, is adopted in [83] to store each subpart feature associated with a vector index. The vector of indices allows fast access to parts that have general similar shapes, and this mechanism accelerates partial retrieval. A Shape-Google framework based on similarity sensitive hashing are also introduced in [45], which offers significant advantages in storage and search complexity. Similarly, subparts are clustered by a K-means clustering algorithm and each cluster groups subparts with similar signatures in a large database[85]. [85] introduces a thesaurus concept from text retrieval and built a 3D shape thesaurus that is a pre-compiled list of terms $\{T_1, \dots, T_n\}$, each of which represents a cluster of similar shapes. The process is depicted in Fig.11. Authors of [85] finally performed a fast global retrieval between the query and the few entries in the thesaurus, and consequently overcame the time complexity problems associated with partial queries in a large database. A set of subpart signatures is also clustered using bag of words (BoWs) in [84, 108], in order to obtain a fixed number of 3D visual words and construct a 3D visual vocabulary to facilitate partial shape retrieval. For an input 3D partial object, its segments are compared with visual words of the vocabulary, and assigned to the most similar visual words. The word histogram of the partial shape is a very sparse vector, and can be directly compared with other histograms of complete objects. In summary, these techniques are able to speed up online retrieval of parts by these efficient data structures.

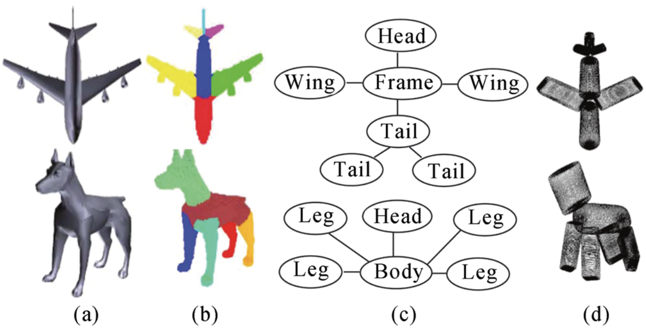

Fig.10. Process of organizing parts proposed by Mademlis *et al.*[88] (a) 3D shapes. (b) Segmented parts. (c) Undirected vertex-attributed graphs. (d) Super-ellipsoid approximation.

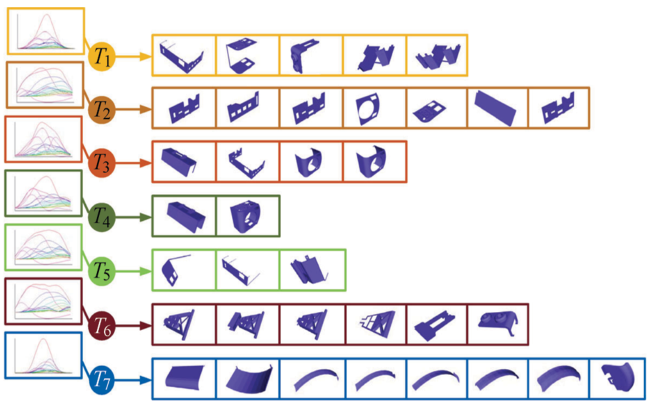

Fig.11. Process of thesaurus creation from [85].

Nevertheless, in contrast to graph algorithms, they discard the interactions between parts.

5.4 Discussion

The above mentioned methods based on preliminary segmentation are effective for matching natural parts with simple topology, and meaningful matches are consistent with human perception. Another merit of these methods is that they decrease the time complexity of partial retrieval by introducing efficient data structures. Segmentation-based retrieval heavily relies on region clustering and graph construction, which are susceptible to noise. Retrieval methods based on Reeb graph are sensitive to small topological changes, for example, part intersections; while mechanical feature based methods utilize fixed and unambiguous relationship built in 3D modeler by designers and hence are robust to topological noise. Other graph-based methods such as common undirected graph, bipartite graph, skeleton, and binary tree, are also easy to be affected by small topological changes. Some methods segment 3D shapes via the rule of greedy clustering, which mainly relies on the convex and concave properties of mesh surface, and accordingly local small disturbance in curvature may be adverse to part definition. Whereas methods based on uniform segmentations and region clustering via image features, are insensitive to small geometrical and topological noises. We theoretically summarize the performances of representative methods in Table 3. Moreover, this category of methods heavily depends on the effectiveness of segmentation. Shape segmentation is far from mature[16]. For example, automatically determining an appropriate number of segments is not an easy problem. Furthermore, final segmentations of some methods are sensitive to deformable models, topologically complex models such as cars, and models with rich geometrical features.

Table 3. Performances of Representative Segmentation-Based Methods

Methods	Rotation Invariant	Need Scaling	Non-Rigid Deformation	Noise
[82], [107-108]	Yes	Yes	Sensitive	Robust
[83], [100]	Yes	Yes	Sensitive	Sensitive
[80], [84-90]	Yes	No	Robust	Sensitive
[91-94]	Yes	No	Robust	Sensitive
[99], [101-103]	Yes	Yes	Robust	Robust

6 View-Based Partial Shape Retrieval

View-based methods do not require segmentation, and commonly rely on generating many views of a 3D object, for example, captured from cameras localized on the unit sphere[109], and matching these views to obtain partial similarity. Because only one or more partial views are used to compare two shapes, we think it is a quite different way to perform partial retrieval. The first step of these methods is to generate a group of 2D images such as silhouette images[3,110-111], depth images[112-117], geometry images[118] with rich orientation information transformed from 3D polygonal data, and panoramic views[119]. The problem of matching a side view to a 3D object is converted to search a most similar image among the group of 2D images. In order to compare two arbitrary images, each image is usually transformed into a feature space by a set of 2D shape descriptors such as 2D Fourier coefficients adopted in [3, 120-121], 2D Wavelet coefficients utilized in [114, 121-122], 2D Zernike moments adopted in [3, 120, 123], 2D Krawtchouk moments adopted by [120, 123], features of Scale Invariant Feature Transform (SIFT) used in [115, 124-125], diffusion tensor field in [111], and Gabor filter used in sketch-based retrieval[126]. Their distances have also been well studied[127]. Therefore, largely owing to many existed 2D descriptors, comparison between two side views can be easily solved by distance metrics of descriptors. Accordingly a key obstacle of these methods changes into the problem how to generate these views. Because the input image can be a random view of a 3D object in any possible direction and position, algorithms should provide views dense enough so as to let one of them correspond with the input accurately. From the viewpoint of time complexity, it is actually unachievable. Fixing viewpoints at several principal axes[114,120] or on a sphere[3,[115], is a tradeoff scheme, which may limit that the input image must be sampled from one of these fixed viewpoints. At the same time, principal axes or symmetry axes of 3D shape need to be computed in advance[116,128]. In order to reduce comparison times, several recent studies[126,129-132] attempt to select the best views for partial retrieval by training a set of image features on a collection of 3D models.

Discussion. The advantages of these approaches lie in their capabilities of supporting 2D image query, 2D sketch query[133], and range scan query. Different from directly inputting a 3D shape, users can easily provide these natural queries consistent with perception. A very recent user-adaptive sketch-based retrieval method[134], first accounts for users' drawing habits, and makes users' sketch more accurately match CAD models by individually weighting a set of visual words based on users' sketching history. These visual words are generated by clustering a set of representative sketches extracted from models in a database. Moreover, there are many successful 2D shape descriptors in the domain of image processing which can be borrowed to match partial views of 3D shape. Most view-based retrieval methods except geometry image based ones are robust to noises including topological noise and geometrical noise, because the 3D shape with small holes and fluctuations on the surface is projected onto 2D planes so that small noises are changed to a few pixels easy to handle by image descriptors such as Fourier transform. However, some view-based methods are sensitive to viewpoint positions, the number of partial views, and their resolutions. They also have a limitation that the process of generating views is not rotation invariant if viewpoints are not dense enough, and sensitive to rigid transformation and non-rigid deformation. A summary about the performances of view-based methods are listed in Table 4.

Table 4. Performances of Representative View-Based Methods

Methods	Rotation Invariant	Need Scaling	Non-Rigid Deformation	Noise
[3], [112], [117], [123], [126], [129-130]	Yes	Yes	Sensitive	Robust
[113-116], [124-125], [131]	No	Yes	Sensitive	Robust
[119], [121-122], [127], [132-133]				
[118]	No	Yes	Sensitive	Sensitive

7 Conclusions

In this article we systematically discussed partial shape retrieval and reviewed the state-of-the-art methods over the last decade. The main contributions behind each class were described in details, and we theoretically compared the advantages and limitations of representative methods of each class. We next survey the evaluation means of partial shape retrieval methods, and discuss possible future directions.

Evaluation. It is difficult to subjectively and mathematically infer which algorithm of partial shape retrieval is better than others. Therefore, the only way of comparison among these algorithms is to adopt a partial retrieval benchmark to evaluate the retrieval performance of these algorithms on a large database, and produce quantitative analysis on retrieval results. Distinct from surface correspondence evaluation, which is based on average geodesic distance error on manual ground-truth landmarks[135], the common metrics used in partial retrieval, dependent on retrieval results, include recall precision values (RP), nearest neighbor (NN), first tier (FT), second tier (ST), E-measure (EM), discounted cumulative gain (DCG), normalized DCG (NDCG), and query rank (QR). Readers can refer to [1, 36] for detailed computation of these metrics. There are different types of datasets for evaluating partial retrieval performance. The main difference among them is the type of input query. According to query ways to search 3D shapes in a target database, we divide them into three types, range image, single partial shape, and hybrid partial shapes. Fig.12 gives five examples of each query type. Table 5 lists several target datasets, and corresponding query type, query set size, target set size, and evaluation metrics.

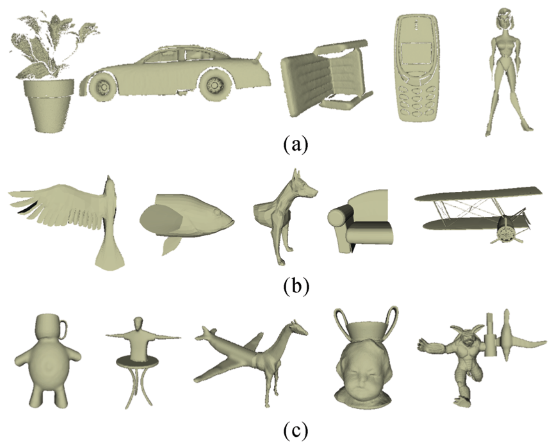

Fig.12. Query input includes three types of shapes. (a) Range image. (b) Single partial shape. (c) Hybrid partial shapes.

Trends. We must face three key challenges: 1) How to improve retrieval accuracy. For local descriptors based methods, it is interesting to propose a sparse, salient, and robust 3D local descriptor like outstanding

Table 5. Statistics of Evaluation Datasets

Dataset	Query Type	Query Set Size	Target Set Size	Evaluation Metrics
SHREC07 Partial matching[136]	(c)	30	400(20)	NDCG
SHREC09 Partial shape retrieval-1[120]	(a)	20	720(40)	NN, FT, ST, EM, DCG
SHREC09 Partial shape retrieval-2[120]	(b)	20	720(40)	NN, FT, ST, EM, DCG
SHREC10 Range scans[137]	(a)	120	800(40)	RP, NN, FT, ST, EM, DCG
SHREC11 Range scans[138]	(a)	150	1 000(50)	RP, NN, FT, ST, EM, DCG
SHREC13 Partial shape retrieval[36]	(a)	7 200	360(20)	RP, NN, FT, ST, QR

Note: The query type belongs to the three common inputs illustrated in Fig.12. In the column of target set size, the amount of the classes of the target database is placed in the bracket.

SIFT in vision, and efficient distance metrics to measure them in feature space. Otherwise, local context information and coding of global information such as symmetry (robust to symmetric flips) should be respected simultaneously. Future methods should also face very poor data with a lot of noise, outliers, missing patches, holes, and also large topological changes. 2) How to recognize and search a shape in a large scene. Recently scene data grow very fast, for example, man-made data in Google 3D Warehouse and data captured by Kinect. These scenes are composed of interest single objects and their spatial contexts, and there is a need to reuse these objects by partial retrieval in the scenes[139-140]. However, many existent 3D geometric descriptors and segmentation methods could not be directly transferred to analyze these scene data, especially incomplete and noisy scan data from Kinect. 3) How to bridge the gap between low-level geometric feature and high-level semantic information. Geometrical description cannot handle objects with man-made rich variations, such as various chairs, which may result in improper partial retrieval. Fully utilizing a priori knowledge about large variations in intra-class shapes is probably a solution.

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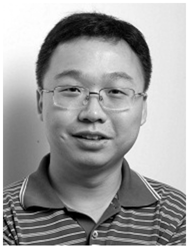
Zhen-Bao Liu received the Ph.D. degree in computer science from the College of Systems and Information Engineering, University of Tsukuba, Japan, in 2009. He is an associate professor at Northwestern Polytechnical University, Xi'an. He was a visiting scholar in the GrUVi Lab of Simon Fraser University in 2012. His research interests include 3D shape analysis, matching, retrieval and segmentation.

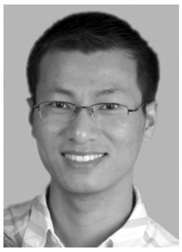
Shu-Hui Bu received the Ph.D. degree in computer science from the College of Systems and Information Engineering, University of Tsukuba, Japan, in 2009. He is an associate professor at Northwestern Polytechnical University, Xi'an. Prior to joining Northwestern Polytechnical University, he was with Kyoto University, Japan. He was a visiting scholar of Simon Fraser University in 2012. His research includes 3D shape analysis, image processing, and computer vision. He has published over 30 journal and conference papers in the related areas.



Kun Zhou is a professor in College of Computer Science and Technology, Zhejiang University, China. He received the B.S. degree and Ph.D. degree in computer science from Zhejiang University. His research concentrates on shape modeling and editing, texture mapping and synthesis, and real-time rendering. He is an associate editor of ACM Transactions on Graphics and The Visual Computer.

Shu-Ming Gao is a professor of the State Key Lab of CAD&CG and College of Computer Science and Technology, Zhejiang University, Hangzhou, China. He received his Ph.D. degree in applied mathematics from Applied Mathematics Department, Zhejiang University in 1990, and was a visiting professor in the Design Automation Lab of Arizona State University and IPK, Germany, in 2001 and 2006, respectively. Currently he served as an associate editor of ASME Trans. JCISE. He is also the committee member of a number of international conferences including ACM SPM, IEEE SMI, PLM, CSCWD, CAD/Graphics, etc. His research interests include CAD, virtual prototyping, collaborative engineering, CAX integration, CAD model retrieval and reuse, engineering informatics, etc.

Jun-Wei Han is a currently a professor with Northwestern Polytechnical University, Xi'an. He received his Ph.D. degree in pattern recognition and intelligent systems from the School of Automation, Northwestern Polytechnical University in 2003. His research interests include computer vision and multimedia processing.



Jun Wu received the B.S. degree in information engineering from Xi'an Jiaotong University in 2001, and the M.Sc. degree and Ph.D. degree both in computer science and technology from Tsinghua University, Beijing, in 2004 and in 2008, respectively. He is currently an associate professor in the School of Electronics and Information, Northwestern Polytechnical University. From 2008 to 2010, he was a research staff in the Intelligent Systems Lab Amsterdam of the University of Amsterdam, the Netherlands. His research interests are in machine learning, multimedia analysis, and multimedia information retrieval.