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Throughput Optimization in Cognitive Radio Networks Ensembling Physical Layer Measurement

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Abstract Wireless networks are developed under the fashion of wider spectrum utilization (e.g., cognitive radio) and multi-hop communication (e.g., wireless mesh networks). In these paradigms, how to effectively allocate the spectrum to different transmission links with minimized mutual interference becomes the key concern. In this paper, we study the throughput optimization via spectrum allocation in cognitive radio networks (CRNs). The previous studies incorporate either the conflict graph or SINR model to characterize the interference relationship. However, the former model neglects the accumulative interference effect and leads to unwanted interference and sub-optimal results, while the work based on the latter model neglects its heavy reliance on the accuracy of estimated RSS (receiving signal strength) among all potential links. Both are inadequate to characterize the complex relationship between interference and throughput. To this end, by considering the feature of CRs, like spectrum diversity and non-continuous OFDM, we propose a measurement-assisted SINR-based cross-layer throughput optimization solution. Our work concerns features in different layers: in the physical layer, we present an efficient RSS estimation algorithm to improve the accuracy of the SINR model; in the upper layer, a flow level SINR-based throughput optimization problem for WMNs is modelled as a mixed integer non-linear programming problem which is proved to be NP-hard. To solve this problem, a centralized $(1-\varepsilon)$-optimal algorithm and an efficient distributed algorithm are provided. To evaluate the algorithm performance, the real-world traces are used to illustrate the effectiveness of our scheme.

Keywords cognitive radio network, wireless mesh network, throughput optimization, centralized algorithm, distributed algorithm, spectrum allocation

1 Introduction

Wireless networks always desire broader coverage and wider transmission bandwidth. The coverage could be enhanced by the wireless mesh networks (WMNs)[1], which support low-cost broadband internet access over large areas in a multi-hop fashion; meanwhile cognitive radios (CRs)[2], which opportunistically utilize the spectrum without interfering with the primary users, will endow the networks with a much wider and dynamic spectrum. Recently, the worldwide implementation of WMNs was witnessed in the commercial project of Google Loon①. The research topics, such as increasing network capacity and guaranteeing QoS in WMNs with cognitive radios, are also widely addressed by the academy society[3].

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①Google Loon Project. http://www.google.com/loon/, Dec. 2014.

For a CR-based WMN, which is illustrated in Fig.1, optimizing the throughput from the internet gateways to the mobile clients is the key concern. This problem in traditional wireless networks is a well-studied problem[4-7]. Most of them treat the core constraint, the interference, as a pairwise relationship, and then, they solve the problem by coloring the conflict graphs. Such simplification, however, ignores the fact that radio interference is inherently accumulative and cannot be accurately represented by pair-wise constraints. As a result, allocations made on top of the graph model could lead to ineffective allocation or unwanted interference. To tackle this, some other work employed the physical interference model (SINR model)[7] into throughput optimization[8-9]. They devoted most of their efforts into solving the non-convex nature of the constraint of SINR.

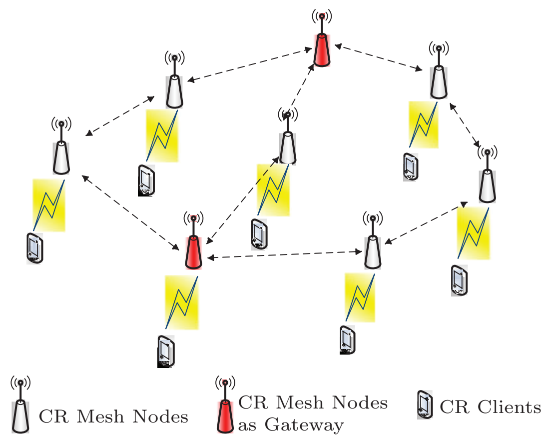

Fig.1. Illustration of CR-based WMN.

Previous studies are not applicable in terms of the throughput optimization in CR-based WMN, as CRs introduce additional complexities and challenges. The primary challenge is *spectrum diversity*. In a traditional 802.11-based WMN, a set of homogeneous spectrums (channels) are always available to every mesh router, while, in a CR-based WMN, each node can access a large number of heterogeneous spectrums. The heterogeneity is conveyed by the so-called feature of spectrum diversity, which implies that different spectrums support different transmission ranges and data rates. It has a significant impact on route selection and spectrum allocation. Meanwhile, CRs also introduce the feature of Non-Continuous Orthogonal Frequency Diversion Multiplex (NC-OFDM), which is considered as the dominant physical layer technology for CRs[3]. By allowing different data flows going from one sender to multiple receivers concurrently, this technology has the potential to improve the throughput of the network, but also incurs higher computational cost for spectrum allocation and link scheduling.

In this paper, we study the throughput optimization problem via spectrum allocation in the CR-based WMNs, like in Fig.1. The major traffic in such a scenario is generated between the *CR clients* and the *gateways*. We want to know how to allocate the spectrum to the potential links so that the aggregated throughput travelling through the network (from clients to the gateways) is maximized. In studying this problem, we take both of the aforementioned challenges into account. The existing studies of throughput optimization based on the SINR model[8-9] neglect the property of spectrum diversity and the fact that the accuracy of the SINR model greatly affects the result of optimization, which is inefficient, as illustrated in our previous work[10]. To this end, we argue that the efficient measurement of RSS of every possible spectrum in every possible link is the key to capturing the spectrum diversity and improving the accuracy of SINR model. Regarding the NC-OFDM feature, it greatly enlarges the solution space and in turn incurs higher computational cost, as it allows either the allocation of multiple ranges of spectrum (channels) into one link, or multiple outgoing links from one node. Thus, we carefully model this problem into a mixed integer non-linear programming problem, and propose efficient centralized and distributed solutions. Our contributions are summarized as follows.

- We present and formulate the throughput optimization problem in CR-based WMN under the SINR model with considering the physical layer features. The problem is formulated as a mixed integer non-linear programming problem, which is generally NP-hard.
- We propose a centralized cross-layer solution. Specifically, it consists of three parts. We first design an efficient SINR measurement algorithm to obtain the accurate SINR. Based on the collected SINR values, a centralized algorithm based on the simplified flow graph is introduced. This algorithm is mainly based on the specification of branch and bound framework. Finally, the approximate result could be further enhanced by a heuristic-based approach. This heuristic-based enhancement could also help greatly improve the efficiency of the whole solution.
- We propose a low-cost distributed solution requiring several rounds of broadcasting and small-scale optimization. Our simulations show that the proposed

algorithm converges after a constant number of rounds (less than six rounds) for most cases.

• A comprehensive simulation study based on the real datasets from the SWIM platform② is performed. The results show that our centralized solution guarantees an approximate ratio of 95% compared with the optimal result, while the distributed solution achieves more than 75% on average.

The rest of this paper is organized as follows. The network model and assumptions are presented in Section 2. Following this, we formally define our problems in Section 3. An approximating centralized solution and a distributed solution are proposed in Sections 4 and 5, consecutively. Section 6 states our experimental methods and results. In Section 7, we introduce related papers. Finally, we conclude this paper in Section 8.

2 System Model and Assumptions

In this section, we develop our network model and assumptions.

2.1 System Model

We consider a wireless mesh network (as shown in Fig.1) with several CR mesh routers $\mathcal{N}$ consisting of internet gateway nodes $\mathcal{N}_G$ and non-gateway nodes $\mathcal{N} \setminus \mathcal{N}_G$. Each mesh router is associated with client nodes $\mathcal{C}_i$. Each node $i \in \mathcal{N}$ senses its environment and finds a set of available spectrum $\mathcal{M}_i$ for the given time (i.e., those bands that are currently not used by primary users), which may not be the same as the available spectrum at other nodes. Without the loss of generality, we assume that the bandwidth of each spectrum band (channel) is denoted as W. Denote $\mathcal{M}$ as the union of all spectrum bands among all the nodes in the network, i.e., $\mathcal{M} = \bigcup_{i\in\mathcal{N}}\mathcal{M}_i$, and each band is identically denoted as m. We also denote $\mathcal{M}_{ij} = \mathcal{M}_i \bigcap \mathcal{M}_j$, which is the set of common bands between nodes i and j.

2.1.1 Interference Model

Different interference models have been extensively studied in the literature, such as Protocol Interferences Model[7], Fixed Protocol Interferences Model[11], RTS/CTS Model[4], and Physical Interference Model (SINR model). We apply the physical interference model here, for its unique advantage to characterize the accumulative feature of interference. The physical interference model is relying on the computation of SINR (signal-to-noise ratio). In this model, concurrent transmissions are allowed and interference (due to transmissions by non-intended transmitter) is treated as noise. A transmission is successful if and only if SINR at the receiver is greater than or equal to a threshold.

The key to computing SINR is to get the values of RSS. Without loss of generality, we assume that every node sends at a uniform power of $P_{\max}$. Considering a transmission from node i to node j on band m, we use P_{ij}^m to denote the power of signal propagation from node i to node j. Based on the above assumptions, we define SINR. When there is interference from concurrent transmissions on the same band, SINR at node of transmission from node i to node j on band m, denoted as s_{ij}^m, is

$$s_{ij}^m = \frac{P_{ij}^m}{N_0 + \sum_{k\in\mathcal{N},k\neq i}\sum_{w\in\mathcal{N},w\neq k,j} P_{kj}^m}.$$

Here, $N_0 = \sigma W$ and σ is the ambient Gaussian noise density.

According to the Shannon capacity formula, the corresponding capacity is always positive. In practice, if SINR is too small, then the achieved capacity will also be very small. In this case, the loss rate of such a link is too high. Thus, we may use a threshold to remove such weak links from consideration. In this regard, we introduce a threshold for SINR, i.e., a transmission from node i to node j on band m is considered successful if and only if $s_{ij}^m > \alpha$.

2.1.2 Throughput Model

CRs deal with spectrums in a dynamic boundary way, rather than the channels with a fixed width. Moreover, the emergence of NC-OFDM technology enables the wireless nodes to combine separate portions of spectrum to form an integrated communication band. Thus, although we previously assumed that the CR nodes sense the spectrum in a fixed width channel manner, each communication link is able to work on multiple channels concurrently. In turn, we introduce the binary indicator a_{ij}^m, which is equal to 1 if node i transmits data to node j on spectrum band m and is equal to 0 otherwise. We also assume that a wireless node cannot transmit and receive in the same channel simultaneously. Although new progress in duplex wireless communication makes such type of communication possible, it requires high computational power. Thus it is not considered here. Once a band $m \in \mathcal{M}_i$ is used

②SWIM platform. http://cs.nju.edu.cn/lwz/swim/swim.html, Mar. 2015.

by node i for transmission or reception, this band cannot be used again by node i for other transmissions or receptions. Formally, we have

$$\sum_{j\in\mathcal{N}} a_{ij}^m + \sum_{k\in\mathcal{N}} a_{ki}^m \leqslant 1.$$

According to the Shannon capacity formula, the capacity of band m on the link, from i to j, denoted as c_{ij}^m, will be

$$c_{ij}^m = W\log_2(1+s_{ij}^m).$$

Thus, the total capacity of the link, from i to j, as c_{ij}, will be

$$c_{ij} = \sum_{m\in\mathcal{M}_{ij}} a_{ij}^m W\log_2(1+s_{ij}^m).$$

2.2 Assumptions

For the algorithm presented in this paper, we assume that the traffic between a node and the gateway nodes could be divided and routed on multiple paths. Thus, we do not have to take specific routing into account. We also assume that the achievable data rate in each link over the whole network is uniformly proportional to the capacity of each link. Therefore, the optimization of the capacity is equal to the optimization of the throughput between clients and gateways. In turn, we will not work on the rate assignment here. In fact, by only adding a set of variables, our solution could achieve rate assignment as well.

3 Problem Formulation

Given a set of mesh nodes and the available spectrum in each node, we ask the question: "How do we allocate the spectrum into the links between nodes so that the aggregated throughput from the gateways to the clients could be maximized?"

Note that the major traffic in WMNs travels between the mesh clients associated with $\mathcal{C}_i$ and $\mathcal{N}_G$. The nodes in $\mathcal{N}\backslash\mathcal{N}_G$ only serve to relay the traffic. Thus, we can model this kind of network as a flow graph $\mathcal{G}$ with $\mathcal{N}_G$ as the source nodes and $\mathcal{C}$ as the flow destination nodes. In graph $\mathcal{G}$, $E_{\rm out}^i$ is the endpoint set of outgoing edges starting with node i, and $E_{\rm in}^i$ is the endpoint set of incoming edges to i. The throughput optimization problem in this situation could be defined as follows.

Definition 1 (Aggregated Throughput Optimization Problem in CR-Based WMNs). *Given a CR-based WMN $\{\mathcal{N},\mathcal{N}_G,\mathcal{C}\}$, and available spectrum set $\mathcal{M}_{ij}$ between nodes i and j, try to find a spectrum allocation vector $\boldsymbol{X} = \{a_{ij}^m | i,j\in\mathcal{N}, m\in\mathcal{M}_{ij}, a_{ij}^m\in\{0,1\}\}$, so that the aggregated throughput between $\mathcal{C}$ and $\mathcal{N}_G$ is maximized.*

In short, we try to find a spectrum allocation vector so that the minimum cut of $\mathcal{G}$ can be maximized.

The aggregated throughput could be formally defined as $\sum_{u\in\mathcal{N},v\in\mathcal{N}_G} f_{uv}$, and the flow in each link must follow the constraint of

$$f_{ij} \leqslant \sum_{m\in\mathcal{M}_{ij}} a_{ij}^m W\log_2(1+s_{ij}^m).$$

Next, we can give the formulation of this problem:

$$\begin{aligned}
\text{Max}\quad & \sum_{u\in E_{\rm in}^v, v\in\mathcal{N}_G} f_{uv}\\
\text{s.t.}\quad & \sum_{k\in E_{\rm in}^i} f_{ki} - \sum_{w\in E_{\rm out}^i} f_{iw} = 0,\\
& \forall i\in\mathcal{N}\setminus\mathcal{N}_G,\\
& f_{ij} \leqslant \sum_{m\in\mathcal{M}_{ij}} a_{ij}^m W\log_2(1+s_{ij}^m), && (1)\\
& \forall i\in\mathcal{N}\bigcup\mathcal{C}, j\in E_{\rm out}^i,\\
& \sum_{k\in E_{\rm in}^i} a_{ki}^m + \sum_{w\in E_{\rm out}^i} a_{iw}^m \leqslant 1, && (2)\\
& \forall i\in\mathcal{N}, \forall m\in\mathcal{M}_{ij}, \mathcal{M}_{ij}\neq\emptyset,\\
& s_{ij}^m = \frac{P_{ij}^m}{N_0 + \sum_{k\in\mathcal{N},k\neq i}\sum_{w\in\mathcal{N},w\neq k,j} a_{kw}^m P_{kj}^m}, && (3)\\
& \forall i\in\mathcal{N}\bigcup\mathcal{C}, i\notin\mathcal{N}_G, j\in E_{\rm out}^i, m\in\mathcal{M}_{ij},\\
& s_{ij}^m \geqslant \alpha a_{ij}^m, && (4)\\
& \forall i\in\mathcal{N}\bigcup\mathcal{C}, i\notin\mathcal{N}_G, j\in E_{\rm out}^i,\\
& m\in\mathcal{M}_{ij}, \mathcal{M}_{ij}\neq\emptyset, && (5)\\
& f_{ij}\geqslant 0,\\
& \forall i\in\mathcal{N}\bigcup\mathcal{C}, i\notin\mathcal{N}_G, j\in E_{\rm out}^i,\\
& m\in\mathcal{M}_{ij}, \mathcal{M}_{ij}\neq\emptyset, && (6)
\end{aligned}$$

where constraint (1) is the flow reservation condition for each relay mesh router. Constraint (2) means the flow in link (i,j) must be smaller than the capacity. Constraint (5) implies $s_{ij}^m > \alpha$ when m is allocated to link (i,j), otherwise this constraint always holds. Constraint (6) is the positive flow condition.

A more useful variant of this problem is fair allocation. Basically, for each node $i\in\mathcal{N}$, demands can be routed in proportion to its aggregate user traffic load $f_{ki}, k\in\mathcal{C}, i\in\mathcal{N}$. In other words, we consider the problem of maximizing such that a fraction of each node demand can be routed. For this problem, the fraction of demand that can be routed is the same for each node. It is defined as follows.

Definition 2 (Demands Fair Fulfillment Problem in CR-Based WMNs). *Given a CR-based WMN* $\{\mathcal{N}, \mathcal{N}_G, \mathcal{C}\}$*, available spectrum set* $\mathcal{M}_{ij}$ *between nodes* i *and* j*, try to find a spectrum allocation vector* $\boldsymbol{X} = \{a_{ij}^m | i, j \in \mathcal{N}, m \in \mathcal{M}_{ij}, a_{ij}^m \in \{0, 1\}\}$*, so that traffic demands of each* $\mathcal{C}_i \in \mathcal{C}$ *can be proportionally fulfilled.*

The proportion could be denoted as λ, and here, $\lambda \in (0, 1]$. We denote the demands of $\mathcal{C}_i$ as d_i, which are the aggregated traffic demands of all clients associated with mesh router i. Then, we can formulate this problem as

$$\begin{aligned} \text{Max} \quad & \lambda \\ \text{s.t.} \quad & \lambda d_i = f_{ki}, \\ & \forall i \in \mathcal{N}, k \in \mathcal{C}_i, \\ & \text{constraints (1)} \sim \text{(6)}. \end{aligned}$$

Note that in both formulations, different targets are defined, but they share the same constraint. From the view of optimization, both problems could be solved uniformly. We denote the formulated problem as P. In the rest of the paper, we will not present the solution for both problems separately. Also, both problems are modeled in the form of a mixed integer non-linear program (MINLP), which is NP-hard in general[12]. As a result, we propose approximation methods to solve them. All the notations used in this paper are summarized in Table 1.

Table 1. Notations

Notation	Description
$\mathcal{N}$	Collection of mesh router nodes
$\mathcal{N}_G$	Collection of gateway nodes in $\mathcal{N}$
N_0	White Gaussian noise
a_{ij}^m	Allocation indicator
f_{ij}	Actual flow on link ij
E_{in}^i	Incoming nodes set of node i
W	Bandwidth of channel m
α	Constant of SINR lower-bound
λ	Proportional fairness constant
N	Number of nodes
$\mathcal{C}$	Collection of client nodes
s_{ij}^m	SINR value on node j when node i is sending to j on band m
c_{ij}	Capacity of link ij
E_{out}^i	Outgoing nodes set of node i
f^m	Central frequency of band m
$P_{\max}$	Transmitting power in each node
η	Path loss component
P_{ij}^m	RSS from i to j on band m

4 Centralized Solution

In this section, we present a centralized solution for both problems defined above. In this solution, we assume that a central server or node is responsible for carrying out the algorithm. The environment information collection is conducted distributively, and propagates to the central server through a common channel which is also responsible for the conveying of allocation results.

4.1 Solution Framework

As defined in Definition 1 and Definition 2, the target problems are NP-hard and could not be optimally solved in polynomial time. One of the general performance guaranteed solutions to this kind of MINLP problem is following the branch-and-bound framework[13]. Previous work on throughput optimization[8], although also using the same optimization technique based on branch-and-bound framework, failed to capture the problem specific features to enhance the algorithm efficiency. The core variable SINR values are derived via PathLoss signal attenuation model[7], which is proved to have low accuracy and will finally harm the optimization results[10]. To tackle above drawbacks, we propose our solutions, whose basic framework could be summarized as follows.

• *Efficient SINR Measurement.* The inaccurate signal propagation models could greatly compromise the optimization results of SINR-based throughput optimization. To tackle this, we introduce how to improve the optimization result through efficient measurement of accurate SINR values based on the compressive sensing method.

• *Flow Graph Construction.* As the flows in the mesh network always transmit between the clients and the gateways, there is no need to search the solution in the cases where the flow goes in a long detour between the clients and the gateways. Hence, in this step, we construct flow graph, so that the variables of E_{in}^i and E_{out}^i will be ensured. This will help us to reduce the problem complexity and the number of variables.

• *MINLP Solving.* We adopt the branch-and-bound framework to obtain the $(1-\varepsilon)$-approximation result for the MINLP. We specify the branch-and-bound framework to solve our problem, including both the upper-bound form of problem and the fast method to find the upper-bound and lower-bound results.

• *Heuristic-Based Enhancement.* After the performance guaranteed allocation via branch-and-bound framework, a simple heuristic is used to enhance the result efficiently. This simple method could serve to balance the trade-off between the efficiency and the accuracy of the optimization.

4.2 Solution Process

As mentioned in Subsection 4.1, our proposed algorithm consists of four steps. In this subsection, we will depict each step in a detailed way.

4.2.1 *Efficient SINR Measurement*

The advantage of our modeling of the problem is the utilization of SINR interference model. However, it also introduces extra complexity over the other model by requiring the transmitting and receiving power at each node. Previous studies usually use PathLoss propagation gain model to estimate the receiving power, which is $P_{ij}^m = Pd_{ij}^{-\eta}$. Here, d_{ij} denotes the distance between node i and node j, and η is the path-loss component. However, this model is far from the real condition. Basically, this model is only valid when the signal is transmitting in line of sight. However, most wireless transmissions may travel through obstacles or endure the multipath effect. Thus, spectrum allocation with the SINR estimated in this model will lead to unpredicted interference, and in turn compromise the whole optimization results. The performance gap between optimization based on propagation model and real data could be found in Fig.3(a). In large area CRNs, the channel condition is even more complex. As a result, the measurement-calibrated SINR is inevitable.

Meanwhile, our optimization scheme requires the SINR in every possible transmission scenarios. Even if we derive the SINR through the RSS of every peer of node, it still requires N^2 time slots to get the measurement result, which is prohibitive in implementation. In our previous work[10], we deal with this problem through a model-based measurement and estimation method. However, it requires the position of each node, and costs the time in the order of $O(N)$. In this subsection, we propose a compressive sensing based solution whose time cost is in order of $O(\log(N))$, which means in the real implementation with on-the-shelf network interfaces, the duration of whole process is in the level of centiseconds.

Our efficient SINR estimation method consists of two components. First, we use a compressive sensing based solution to estimate the RSS between nodes in one spectrum. Then, we introduce how to extend the result in one spectrum into the others.

1) *Single Spectrum Condition*

According to the SINR model, the signal strength is linearly additive. Thus, a linear combination of measurements could be regarded as a linear system $\boldsymbol{Y} = \boldsymbol{AX}$, where $\boldsymbol{Y} \in \boldsymbol{R}^N$ is the measurement result in each node, $\boldsymbol{X}\boldsymbol{R}^{N\times N}$ is the signal strength, and $\boldsymbol{A} \in \{0,1\}^{M\times N}$ is the sending schedule. According to Cramer's rule[14], the linear system has a unique solution if only $M = N$. But our target is to generate a matrix $\boldsymbol{A}$ with $M << N$. In this situation, the linear system could be resolved with the prevalent Compressive Sensing[15] concept.

The success of the solution depends on two crucial components. The first one is the generation of the measurement matrix with good RIP. Here, good RIP, according to the theorem in [16], refers to that RIC (restricted isometry constant) is smaller than $\sqrt{2}-1$. Further, we should also find representation basics in which the signal strength matrix could be represented in the form of a K-sparse matrix. Note that in our problem, we deal with a matrix rather than a vector, which is concerned in the previous work. In the context of matrices, low rank is analogous to sparsity because the spectrum formed by the singular values of a low-rank matrix is sparse. Thus, in our problem, the K-sparse matrix implies that the rank of the matrix is K.

Assume that the signal matrix is K-sparse in a certain domain, formally stated as:

$$\boldsymbol{X}' = \boldsymbol{\Psi X},$$

where $\boldsymbol{X}'$ is a K-sparse matrix. Then we have

$$\boldsymbol{Y} = \boldsymbol{\Phi\Psi X}',$$

where $\boldsymbol{\Phi}$ is the measurement matrix. Based on these formalizations, we now proceed to the determination of the measurement matrix and representation basics.

Note that, in the matrix, most of the elements are in fact close to but not equal to zero. This situation requires us to carefully drop off some elements to make the matrix sparse. With all these considerations, we apply the singular value decomposition here.

Simply stated, our $N \times N$ matrix could be decomposed such that:

$$\boldsymbol{X} = \boldsymbol{U\Sigma V}^{\mathrm{T}}, \tag{7}$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are the $N \times N$ unitary matrices (i.e., $\boldsymbol{UU}^{\mathrm{T}} = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{U} = \boldsymbol{I}$), and $\boldsymbol{\Sigma}$ is an $N \times N$ diagonal matrix containing the singular values. The rank of a matrix is the number of linearly independent rows or columns, which equals the number of nonzero singular values of $\boldsymbol{\Sigma}$.

In our method, the measurement matrix is a binary matrix, with each row as the sending plan of all nodes in one time slot. The number of rows represents the number of time slots used to perform measurement. In addition, because the representation basics are usually composed of an orthonormal matrix, the restricted isometry constraints of matrix $\boldsymbol{\Phi\Psi}$ are the same as those of $\boldsymbol{\Phi}$. Thus, our target is to find a binary matrix, which has good RIP and a small number of rows.

As aforementioned, the measurement matrix in CS is usually drawn from a random matrix whose entries are i.i.d. Guassian variables complying to $\mathcal{N} \sim (0, 1/M)$[17]. In our application, a binary measurement matrix is required. A simple way to generate a binary matrix is to generate each entry of $\boldsymbol{\Phi}$ complying to the Bernoulli distribution with success probability p. According to the proof in [18], this kind matrix bears good RIP *w.h.p.*

According to [19], the perfect recovery to a matrix is equal to solving the following question:

$$\min rank(\boldsymbol{X}) \text{ s.t. } \boldsymbol{Y} = \boldsymbol{\Phi X}.$$

However, minimizing the rank would be rather hard to solve. Thus, in [19], the authors give out an equivalent form of problem:

$$\min ||\boldsymbol{X}||_* \text{ s.t. } \boldsymbol{Y} = \boldsymbol{\Phi X},$$

when $\boldsymbol{\Phi}$ has a good RIP. Here, $||\boldsymbol{X}||_*$ is the nuclear norm, which is the sum of all elements of $\boldsymbol{\Sigma}$ in (7).

In summary, a central server is needed here to generate the random binary measurement matrix. Each node performs the sending and measuring according to the measurement plan, as defined by the measurement matrix, and collects the RSS in every time slot. The recovery is performed in the central node.

2) *Multiple Spectrum Condition*

Now we introduce how to extend the measurement result of a channel to other channels over the same link. A frequency related signal propagation model which is specified for cognitive radios, is introduced in [20]. The formulation is:

$$PL_{ij}^m = 32.4 + 20\log_{10}(f_m d_{km}) + S_{d_{km}} \text{ dB}, \quad (8)$$

where f_m is the frequency of channel m, and S is the channel fading. The signal attenuation is represented in unit of dB. According to [20], the channel fading is hardly related to the frequency band. Thus, in the following discussion, the channel fading is only represented by a variable $S_{d_{km}}$.

With (8), we can have following results. Suppose p_{f_1} is the received power at n_2 when the transmission happens using carrier frequency f_1, and similarly p_{f_2} is defined. Then, we can obtain:

$$p_{f_1} - p_{f_2} = 20\log\left(\frac{f_2}{f_1}\right). \quad (9)$$

Without loss of generality, assume $f_1 < f_2$. The equation shows that if either p_{f_1} or p_{f_2} is known, the other can be inferred. With this equation, a measurement on one spectrum could derive the RSS of another spectrum in the same link.

It is worth to mention that (9) could fail if there exists severe channel selective fading which is caused by obstacles and multipath effect. In this case, we can use the two-channel measurement to firstly examine whether the channel selective fading is large enough. If it is, per-channel measurement could be conducted to improve the accuracy. Otherwise, we can still use (9) to perform the SINR measurement.

4.2.2 Flow Graph Construction

The flow graph, which defines the outcoming node set E_{out}^i of node i, and its ingoing node set E_{in}^i, plays an important role in our algorithm. The main reason we construct the flow graph is to reduce the problem complexity. Note that, if we do not specify E_{out}^i and E_{in}^i, it means $E_{\text{out}}^i = E_{\text{in}}^i$ including all adjacent nodes of i. This will increase the number of variables and greatly increase the complexity of the problem.

We construct the flow graph based on the intuition that the spectrum would be assigned on the path which connects the gateway with the best spectrum utilization. The details of the algorithm are presented in Algorithm 1.

For this algorithm, we have the following theorem.

Theorem 1. *Let $F_{\max}$ be the maximum possible throughput of network $\mathcal{N}$, and F_G be the maximum possible throughput of the flow graph G, which is constructed with Algorithm* 1 *on $\mathcal{N}$. If we have a correct metric to mark the spectrum utilization in the link, then we have $F_G = F_{\max}$.*

Proof. We prove this via Reductio ad absurdum. Assume we have $F_{\max} > F_G$. Then, there exists one link from node i to node j, when the hop count of i is smaller than the one of j. Let the graph with $F_{\max}$ be noted as G'. Then, we have $F_{G'} = F_{\max}$. From the flow graph's view, the link (i, j) must take extra flow l from i to j, and then to the gateways. Due to the NC-OFDM feature, we can allocate multiple channels to one link. Then, this extra flow could also be realized in the path from i to the gateway g. It is trivial that the path from i to j, and then to the gateway, is longer than the path from i to g with minimum spectrum cost. In other words, it takes less spectrum resources to realize the flow l. Then, there must be one graph G'' which is not G and $F_{G''} > F_G$. This contradicts $F_G = F_{\max}$. Thus, we have $F_G = F_{\max}$. $\square$

Algorithm 1. Flow Graph Construction

Require: available band $\mathcal{M}_i$
Ensure: E^i_{in} and E^i_{out} of node i

Assign the value of all links between nodes as a spectrum utilization metric
Delete the links with the value smaller than α
Perform a Floyd-Warshall algorithm to find the shortest path between all node pairs
for $\forall i \in \mathcal{N}$ **do**
  Let $value(i)$ be the sum of all values in its shortest path
end for
for $\forall i \in \mathcal{N}$ **do**
  for $\forall j \in \mathcal{N}$ **do**
    if $value(i) > value(j)$ & $value_{ij} < \alpha$ **then**
      Put i into E^j_{in} and j into E^i_{out}
    end if
  end for
end for
Put the sending nodes into E^i_{out}

This means that the edge trimming performed by Algorithm 1 will not affect the throughput optimization result if we have the correct metric. However, it is hard to get the correct metric due to that the spectrum utilization in one link is affected by nearby links. In our implementation, we use N_0/RSS_{ij} as the spectrum utilization metric. Because of the utilization of the Floyd-Warshall algorithm, the time complexity of Algorithm 1 is $O(N^3)$.

4.2.3 MINLP Solving

As stated before, both targeting problems are NP-hard. Thus, it could only be solved in an approximate way. We try to follow the branch-and-bound framework[13] and get a $(1-\varepsilon)$-approximate result. This framework requires an upper-bound form of original problem and a lower-bounded one. With both forms, this framework solves the problem by splitting the solution space into multiple small sets, and it gets the best results in all branches. We follow this framework by first trying to relax those non-linear conditions so that the relaxed form of the problem could serve as the upper bound in the branch-and-bound framework. We also provide a method to find a feasible solution to the problem, which could serve as the lower bound.

Upper Bound. There are two non-linear conditions: (2) and (4). For the nonlinear term $\log_2(1+s^m_{ij})$, we employ convex hull linear relaxation that contains $\log_2(1+s^m_{ij})$. Suppose that we have the bounds for s^m_{ij}, i.e., $(s^m_{ij})L \leqslant s^m_{ij} \leqslant \overline{s^m_{ij}}$. We introduce a variable $t^m_{ij} = \log_2(1+s^m_{ij})$ and consider how to get a linear relaxation for t^m_{ij} . The curve of $t^m_{ij} = \log_2(1+s^m_{ij})$ can be bounded by four segments (or a convex hull), where segments 1, 2, and 3 are tangential supports and segment 4 is the chord. In particular, the three tangent segments are tangential at points $(\underline{s^m_{ij}}, \log_2(1+\underline{s^m_{ij}}))$, $(\beta, \log_2(1+\beta))$, and $(\overline{s^m_{ij}}, \log_2(1+\overline{s^m_{ij}}))$. Clearly, s^m_{ij} is upper-bounded by P/N_0 and is lower-bounded by 0. Thus, $\overline{s^m_{ij}} = P/N_0$ and $\underline{s^m_{ij}} = 0$. For simplicity, we take $\beta = P/(2N_0)$. Segment 4 is the segment that joins points $(0,0)$ and $(\beta, \log_2(1+\beta))$ and segment 5 is the one between $(\beta, \log_2(1+\beta))$ and $(P/N_0, \log_2(1+P/N_0))$. The convex region (as shown in Fig.2), defined by the five segments, can be described by the following linear constraints:

$$t^m_{ij} - s^m_{ij} \leqslant 0, \tag{10}$$

$$(1+\beta)t^m_{ij} - s^m_{ij} \leqslant (1+\beta)(\log_2(1+\beta)-1)+1, \tag{11}$$

$$t^m_{ij}(1+P/N_0) - s^m_{ij} \leqslant (1+P/N_0)(\log_2(1+P/N_0)-1)+1, \tag{12}$$

$$\beta t^m_{ij} - \log_2(1+\beta)s^m_{ij} \geqslant 0, \tag{13}$$

$$(\beta/2)t^m_{ij} - \log_2(1+P/N_0) \geqslant \log_2(1+\beta)s^m_{ij} + (P/N_0)(1+\beta). \tag{14}$$

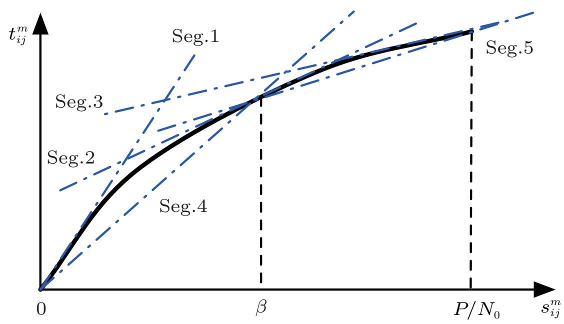

Fig.2. Convex hull of $t^m_{ij} = \log_2(1+s^m_{ij})$. Seg.: Segment.

As a result, the nonlinear constraint (2) could be rewritten as

$$f_{ij} \leqslant W \sum_{m \in \mathcal{M}_{ij}} a_{ij}^m t_{ij}^m. \tag{15}$$

For the SINR constraint in (4), we can rewrite it as

$$N_0 s_{ij}^m + s_{ij}^m \sum_{k \in \mathcal{N}, k \neq i} \sum_{w \in \mathcal{N}, w \neq k, j} a_{kw}^m P_{kj}^m - P_{ij}^m = 0. \tag{16}$$

We put constraints (1), (3), (5), (6) and (10)~(16) together to form a convex optimization problem $\overline{P}$. The value of the target function of its solution, denoted as $\overline{Z}$, could serve as the upper bound. However, solving a linear programming still costs too much, especially considering the branch-and-bound framework will conduct these several rounds. To tackle this, we apply the duality method here. Hence, a feasible solution, which only requires linear running time to find one, to the duality form of the LP could serve as the upper bound.

Lower Bound. The lower-bound solution of the problem, noted as $\underline{P}$, is the solution that satisfies all the constraints in P. Any feasible solution to P could serve as $\underline{P}$. Here, we provide a fast method to find a feasible solution. Clearly, the vector $\{a_{ij}^m = 0, \forall i, j, m\}$ is a feasible solution. We can find a good feasible solution by starting from the solution to $\overline{P}$. We sequentially change the non-zero elements of $\overline{P}$ to 0 until it satisfies all the constraints in P. The value of the target function for the $\underline{P}$, is denoted as $\underline{Z}$.

Algorithm. The main branch and bound algorithm designed for our problem could be depicted as shown in Algorithm 2.

Algorithm 2. Main Branch & Bound Algorithm

Require: available band $\mathcal{M}_i$, $\mathcal{M}_{ij}$

Ensure: channel assignment vector $\{a_{ij}^m\}$

1: Set $k = 1$, and add P into the problem list. Pick the first variable in $\{a_{ij}^m\}$ as a_1

2: Get $\underline{Z}^1$ and $\overline{Z}^1$

3: **while** $\underline{Z}^k < (1-\varepsilon)\overline{Z}^k$ **do**

4: Divide the problem P^k with the variable a_k and get two problems $P_1^k(a_k = 0)$ and $P_2^k(a_k = 1)$

5: Get $\overline{Z_1^k}$, $\overline{Z_2^k}$, $\underline{Z_1^k}$, $\underline{Z_2^k}$

6: Set $\underline{Z} = \max(\underline{Z_1^k}, \underline{Z_2^k})$

7: Remove all the P' in the problem list with $\underline{Z'} < \underline{Z}$

8: Add P_1^k and P_2^k into the problem list

9: $k = k + 1$

10: Sequentially select next variable in $\{a_{ij}^m\}$ as a_k

11: Set P^k as the problem with the largest $\overline{Z}$ in the problem list

12: **end while**

13: Set $\{a_{ij}^m\}$ as $\underline{P}^k$

By this algorithm, we get a $(1-\varepsilon)$-approximation result, where ε determines how many rounds we must perform to approach the optimal result.

4.2.4 *Heuristic-Based Enhancement*

Theoretically, in the branch-and-bound process, we could find an almost optimal result if we set $\varepsilon = 0$. However, according to our experiment, it is better to set $\varepsilon = 0.2$, if we want the algorithm terminating at an acceptable time. Thus, it left us an opportunity to enhance results with some heuristics. We propose a problem-specific heuristic to quickly enhance the result.

The basic idea of this enhancement process is to reallocate the channels among the minimum cut and its neighbours in the network, so that the maximum flow of the existing assignment will be enhanced. This process will be performed for multiple rounds until the flow value cannot be increased. This process is also performed in a centralized manner, as depicted in Algorithm 3.

This algorithm will terminate after performing a certain number of rounds of some maximum flow methods. If one adopts the well-known Edmonds-Karp algorithm[21], then the complexity is $O(N^3)$.

4.3 Discussion

We have to deal with the efficiency problem of branch-and-bound based solution. As a well-known MINLP solution framework, it requires several rounds of solving the linear programming. Consequently, this will incur severe computational cost and long computation time. This framework has been applied in solving spectrum allocation problem[8], where, as the authors claimed, this method could only serve as the benchmark solution. In our centralized solution, we further improve the efficiency of this framework in two folds. 1) We reduce the number of variables, and thus reduce the computational complexity. 2) We use a heuristic-based enhancement algorithm to improve the results computed by branch-and-bound based solution. This heuristic-based enhancement is very efficient and effective. This algorithm also endows us with the opportunity that we could start the enhancement from a comparatively low-approximation point (e.g., $\varepsilon = 0.2$), as the computational complexity grows exponentially with $(1-\varepsilon)$. Thus, with the heuristic-based enhancement, we further improve the efficiency of the centralized solution.

Algorithm 3. Post Allocation Adjustment

Require: available band $\mathcal{M}_i$, $\mathcal{M}_{ij}$

Ensure: channel assignment vector $\{a_{ij}^m\}$

1: Perform a maximum flow algorithm on the assigned flow network G_f and get the maximum flow value f_v and the minimum cut set $S_{\min}$

2: **for** $\forall ij \in S_{\min}$ **do**

3: $\quad \rho_i = |\sum_{k\in E_{\text{in}}^i} c_{ki} - \sum_{w\in E_{\text{out}}^i} c_{iw}|$

4: $\quad \rho_j = |\sum_{w\in E_{\text{out}}^j} c_{jw} - \sum_{k\in E_{\text{in}}^j} c_{kj}|$

5: $\quad$ **for** $\forall m \in \mathcal{M}_{ij}$ and $a_{ij}^m = 0$ **do**

6: $\quad\quad$ Backup original value and let $x_{ki}^m = 0, x_{lj}^m = 0, x_{it}^m = 0, x_{jw}^m = 0, \forall k \in E_{\text{in}}^i, w \in E_{\text{out}}^j, t \in E_{\text{out}}^i, l \in E_{\text{in}}^j$ and $a_{ij}^m = 1$

7: $\quad\quad$ Recompute $c_{ki}^{'}, c_{jw}^{'}, c_{it}^{'}, c_{lj}^{'}, \forall k \in E_{\text{in}}^i, w \in E_{\text{out}}^j, t \in E_{\text{out}}^i, l \in E_{\text{in}}^j$

8: $\quad\quad \rho_i^{'} = |\sum_{k\in E_{\text{in}}^i} c_{ki}^{'} - \sum_{w\in E_{\text{out}}^i} c_{iw}^{'}|$

9: $\quad\quad \rho_j^{'} = |\sum_{w\in E_{\text{out}}^j} c_{jw}^{'} - \sum_{k\in E_{\text{in}}^j} c_{kj}^{'}|$

10: $\quad\quad$ **if** $\rho_i^{'} + \rho_j^{'} \geqslant \rho_i + \rho_j$ **then**

11: $\quad\quad\quad$ Restore the original values of $a_{ij}^m, x_{ki}^m, x_{lj}^m, x_{it}^m, x_{jw}^m, \forall k \in E_{\text{in}}^i, w \in E_{\text{out}}^j, t \in E_{\text{out}}^i, l \in E_{\text{in}}^j$

12: $\quad\quad$ **end if**

13: $\quad$ **end for**

14: **end for**

15: Perform a maximum flow algorithm on the assigned flow network G_f with new $\{a_{ij}^m\}$ and get the maximum flow value $f_v^{'}$ and the minimum cut set $S_{\min}$

16: **if** $f_v^{'} > f_v$ **then**

17: $\quad f_v = f_v^{'}$

18: $\quad$ goto 2;

19: **end if**

5 Distributed Solution

The centralized solution is not suitable for conditions without a strong centralized server or it is hard to find a common channel to transmit the control message. Besides, the optimization process is relatively costly.

In this section, we present a low-cost, fast convergence distributed solution. The basic idea is as follows. Our optimization objective is a global one, and could not be achieved locally. Thus, to achieve the best effort, we use a local heuristic, which is maximizing the aggregated flow travelling through the current node. The most sophisticated part of local channel assignment is to resolve the conflict locally. We manage to do this via an iterative neighbour consensus process.

The distributed algorithm consists of two phases, initialization and adjustment. The former produces an initial assignment with the best try. Then, the initial assignment will be improved in the second phase.

5.1 Initialization

In the initialization phase, we first perform the efficient SINR measurement algorithm and flow graph construction. In this way, each node i is able to get $P_{wi}^m, \forall w \in \mathcal{N}, m \in \mathcal{M}_i$ and $E_{\text{in}}^i, E_{\text{out}}^i$. Then, each node shares the information of $E_{\text{in}}^i, E_{\text{out}}^i$ and $\{P_{wi}^m\}$ with its one-hop neighbour set(v_1^i) by performing one round of broadcasting. After this, each node is able to compute the value of $\widehat{s_{ki}^m}, \forall k \in v_1^i$, which is defined by the following equation:

$$\widehat{s_{ki}^m} = \frac{P_{ki}^m}{\sum_{w\in\mathcal{N}} P_{wi}^m + \sigma W}.$$

Then, each node broadcasts $\{\widehat{s_{ki}^m}\}$ to its neighbor. After this process, node i is able to compute its probability to use channel m in any outgoing edges, denoted by u_i^m, as defined by the following equation:

$$u_i^m = \min(1, \max(\{\widehat{s_{iw}^m}/\alpha, w \in E_{\text{out}}^i\})).$$

u_i^m will also be shared by neighbors. The expected throughput of each channel is formally defined as:

$$\widehat{c_{kl}^m} = \frac{P_{kl}^m}{\sum_{w\in v_1^l} u_w^m P_{wl}^m + \sum_{w\in\mathcal{N}\setminus v_1^l} P_{wl}^m + \sigma W}.$$

Another broadcast of $\widehat{c_{kl}^m}$ is required so that each node gets the expected throughput of each channel in all its incoming and outgoing edges.

5.2 Adjustment

In this subsection, we will present the method to enhance the performance of the initial assignment. In this process, each node uses the collected information to solve the following local optimization problem:

$$
\begin{aligned}
\max \quad & \min\left(\sum_{k\in E_{\text{in}}^i} f_{ki}, \sum_{w\in E_{\text{out}}^i} f_{iw}\right)\\
\text{s.t.} \quad & f_{kl} = \sum_{m\in\mathcal{M}_{kl}} a_{kl}^m c_{kl}^{\hat{m}},\\
& \sum_{k\in E_{\text{in}}^i} a_{ki}^m + \sum_{w\in E_{\text{out}}^i} a_{iw}^m \leqslant 1.
\end{aligned}
$$

Solution space for one node assignment is very small; thus, it can be solved efficiently. After grabbing the assignment results in node i, denoted as a_{ij}^{mi}, each assignment should be the consensus result of both endpoints. Formally, $a_{ij}^m = a_{ij}^{mi} \times a_{ij}^{mj}$. This requires another broadcast of the assignment results.

The distributed algorithm is summarized in Algorithm 4.

Because each node has limited number of channels, and each round will make at least one assignment, this algorithm will eventually converge. Regarding the computational complexity, as the algorithm only requires the nodes communicate with the neighboring nodes in a constant number of rounds, the complexity of the distributed algorithm is only $O(N^2)$.

6 Evaluation

In this section, we introduce the network simulation which examines the effectiveness of our algorithms. We first present our simulation target and settings, and then it follows with numerical results.

6.1 Experiments Settings

To illustrate the effectiveness of our algorithms, we have to provide the results of the following items:

- the performance of efficient SINR measurement in terms of flow enhancement;
- the effectiveness of heuristic enhancement process in terms of flow enhancement;
- the performance of our centralized solution and distributed solution;
- the cost of our distributed solution and heuristic enhancement in terms of convergent rounds.

For one set of network settings, we have to give out the mesh node set $\mathcal{N}$, the gateway node set $\mathcal{N}_G$, the available channel set $\mathcal{M}_i$ in each node, and finally, the receiving power from node i to node j in band m as P_{ij}^m.

Our network settings are based on the trace collected from SWIM platform. In our experiment, we treat measured points as user locations, and use the signal strength in each beacon as the signal or interference power. We associate each user with the AP carrying the strongest signal. This produces 151 APs with at least one user associated.

Clearly, this dataset is not cognitive radio data. However, we can use the power and position information to generate our network scenarios.

Generally, we extract network scenarios in this way: randomly pick n nodes from our trace. We also set the total operating bandwidth to approximately 2.4 GHz, with m orthogonal channels of 20 MHz, which are the general settings in IEEE 802.11. We use the trace data to generate $\{P_{ij}^m\}$. We assume that the trace data of the RSSI (receiving signal strength index) in node j from node i in band l is P_{ij}^l, where l is the band centered at 2.4 GHz. In Wi-Fi, the RSSI is enclosed

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Algorithm 4. Distributed Algorithm

Require: incoming links E_{in}^i, outgoing links E_{out}^i, and the available spectrum $\mathcal{M}$

Ensure: assignment of spectrum for each link

1: Share the assignment result with two-hop neighbours via two rounds of broadcasting

2: Each node recomputes the expected throughput of each unassigned channel m in each of the incoming links $\tilde{c}_{kl}^m = \frac{P_{kl}^m}{\sum_{w\in v_1^l \cup v_2^l} a_{wl}^m P_{wl}^m + \sum_{w\in\mathcal{N}\setminus(v_1^l \cup v_2^l)} P_{wl}^m + \sigma W}$

3: Recompute the aggregated throughput of node i under the current assignment $\{a_k^i, k \geqslant 1\}$, which is $\min(\sum_{k\in E_{\text{in}}^i} f_{ki}, \sum_{w\in E_{\text{out}}^i} f_{iw})$. If the aggregated throughput is not increased, use $\{a_{k-1}^i\}$ as the final assignment of node i and terminate the algorithm

4: Based on $\tilde{c}_{kl}^m$, resolve the optimization problem LP_1 again for all the unassigned channel m, without modifying the existing assignment where $a_{ij}^m = 1$

5: Make a consensus for the assignment from both endpoints

6: **if** there are channels or links not assigned **then**

7:     Goto step 1

8: **end if**

in the packet. Thus, we are not able to get P^l_{ij} when a packet transmitted from i to j cannot be decoded. However, the receiving signal in j from i can still be an interference. As a result, we have to derive P^l_{ij} for each node pair, which could be computed in this way:

1) For a given P^l_{iw}, compute $\widehat{P^{lw}_{ij}} = (\frac{d_{iw}}{d_{ij}})^{-\alpha} P^l_{iw}$.

2) Compute $\overline{P^l_{ij}}$ using the average value of $\widehat{P^{lw}_{ij}}$.

After the fetch of P^l_{ij}, $\{P^m_{ij}\}$ in other bands could be derived through (9).

The available channels in each node are constrained by the PU nodes. We randomly deploy a certain number of PU nodes with assigned working channels. All the nodes within the communication range of PU could not share the same channels.

In this way, one simulation scenario is generated. We generate 200 scenarios to perform a statistical performance evaluation.

6.2 Numerical Results

Based on the generated scenarios, we perform a statistic evaluation on the performance and cost of our algorithms. The details of 200 generated scenarios are listed in Table 2. We also set the network parameters, like α, σ, to commonly used values as shown in Table 3. Without the loss of generality, we set ε to 0.05.

Table 2. Generated Scenarios

Number of Nodes	Number of Scenarios	Number of Total Spectrums	Number of PU	Number of PU's Spectrums
5	20	10	3	4
10	30	10	5	4
15	30	20	5	6
20	40	20	10	6
25	40	40	10	10
30	40	40	10	10

Table 3. Simulation Settings

Parameter	Value
α	3.0
η	4.0
σ	−4.8

Note that, we obtain the optimal solution via branch-and-bound method with $\varepsilon = 0$, which is too costly in scenarios with over 20 nodes. Hence, if there is no specific explanation, the comparison to the optimal result is performed upon the generated scenarios with nodes numbering no larger than 20.

6.2.1 *Performance of SINR Measurement*

Compared with previous work, the major difference is that we use an efficient SINR measurement procedure instead of the power propagation model to measure the interference. We conduct a statistical evaluation of its effectiveness over the propagation model based on the positions. We adopt the centralized solution based on the interference computed from the propagation model and the one with efficient SINR measurement procedures on all our generated scenarios. The cumulative results are shown in Fig.3(a). From Fig.3(a), we can tell that the efficient SINR measurement procedure helps the centralized solution to enhance its performance and mitigate the performance degradation introduced by the inaccurate power propagation gain model.

Regarding the accuracy of (9), we use two USRP devices to perform a validate experiment. The data are collected in both 900 MHz and 5.7 GHz. One device transmits a packet in power of 20 dBm (100 mW), while the other device measures the receiving power in a certain distance away. Five sets of communications links are used to collect RSS. We use the measurements in both 900 Mhz and 5.7 GHz as the ground-truth, and compare the predicted RSS in both bands computed by (9). The results are shown in Fig.3(c), from which the predict lines (5.7 GHz-Cal and 900 MHz-Cal in the graph) are very close to the averaged measurement lines (dot-lines named as 5.7 GHz and 900 MHz). We can also see that the values of the RSS measured in 5.7 GHz vary much larger than the ones measured in 900 MHz. A possible explanation is that 5.7 GHz has been used for 802.11a/ac communication, while the interference in 900 MHz is much less. Note that the unit used to measure the RSS is dBm, thus the gap between the predicted RSSs and the measured ones is in fact larger in unit of W. In calculating the SINR, the predicted values should be used carefully. In summary, (9) could serve to predict the RSS in different bands with acceptable accuracy.

6.2.2 *Performance of Centralized Solution*

To better understand the performance of our comprehensive centralized solution, we compare our algorithm not only with the optimal results, but also with the algorithms introduced in Section 5. They are denoted as Mobi[22] and B&B[8], respectively. ε is set to 0.05. As we have mentioned before, neither of the algorithms is designed to allocate multiple channels to one link; thus, we change the flow graph by adding several

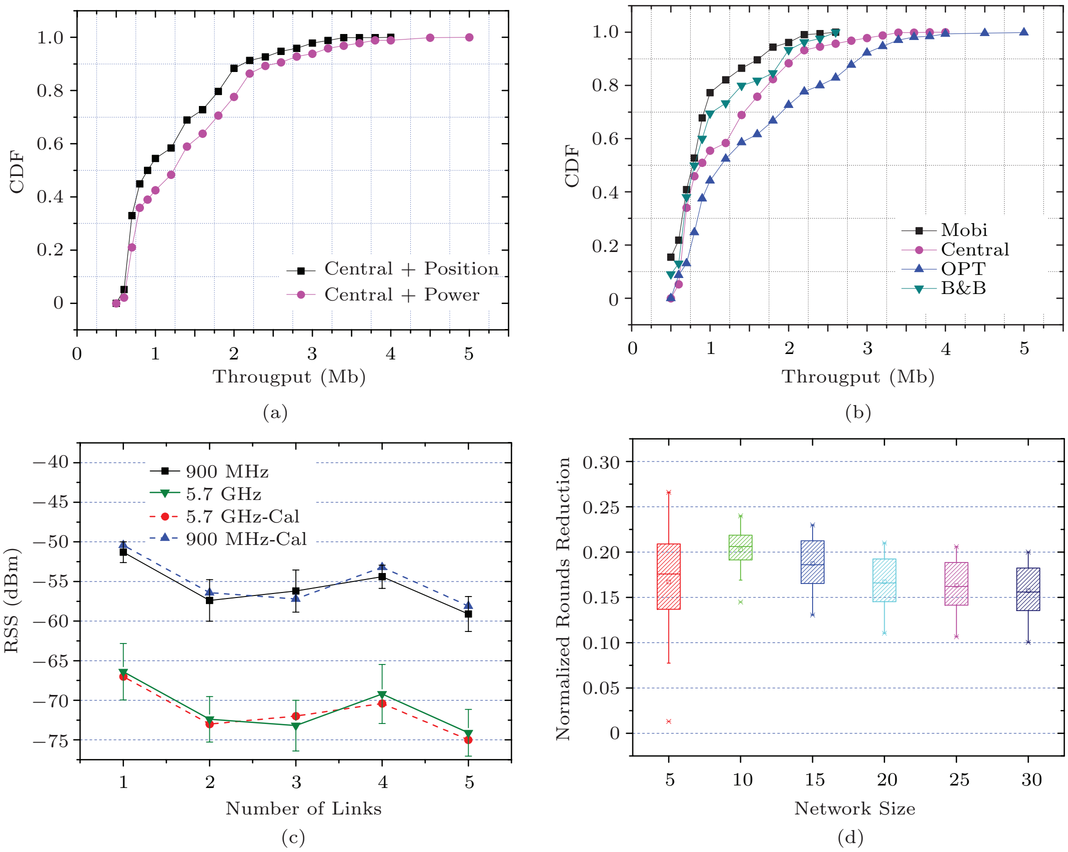

Fig.3. Performance of centralized solution. (a) RSS collection vs propagation model. (b) Centralized solution vs optimal solution and previous solutions. (c) Accuracy evaluation using cross channel RSS estimation in (9). (d) Computation cost reduction between branch-and-bound only algorithm and our centralized algorithm.

parallel virtual links, and combine them when computing the accumulative throughput. To obtain the optimal results, this group of experiments is not conducted upon scenarios with over 20 nodes.

To illustrate the efficiency of our solution compared with a branch-and-bound only algorithm, we introduce a set of experiments, whose results are shown in Fig.3(d). This graph shows that compared with the centralized only algorithm, our algorithm could greatly reduce the computational cost with approximately 80% in average. The results in Fig.3(b) show that our algorithm outperforms both algorithms in the control group. There is still a margin between ours and the optimal results.

We also evaluate the effectiveness of heuristic-based enhancement, and we compare the performance of the centralized solutions with/without enhancement. Both algorithms are conducted upon all scenarios. The results are shown in a CDF form in Fig.4(b). We can see that the heuristic-based enhancement improves the overall throughput in a considerable margin.

In terms of the cost of the heuristic enhancement, we use the benchmark of the number of iterations for the enhancements conducted. The results can be found in Fig.4(d). We can see that, regardless of the network size, the enhancement will be terminated in no more than seven rounds. In particular, for most of the cases, the enhancement will be finished in around 3∼6 rounds.

6.2.3 *Performance of Distributed Solution*

We evaluate our distributed solution by comparing it with the centralized solution and the optimal results. They are all conducted in scenarios with less than 20 nodes. We can see the results in Fig.4(a). The average result shows that the distributed solution achieves approximately 78% of optimal results.

Regarding the overhead of the distributed solution, we measure its convergence speed in terms of the number of iterations. Our simulation results, illustrated in Fig.4(c), show that the proposed distributed algorithm converges before seven iterations. For most cases, the algorithm could terminate within four rounds.

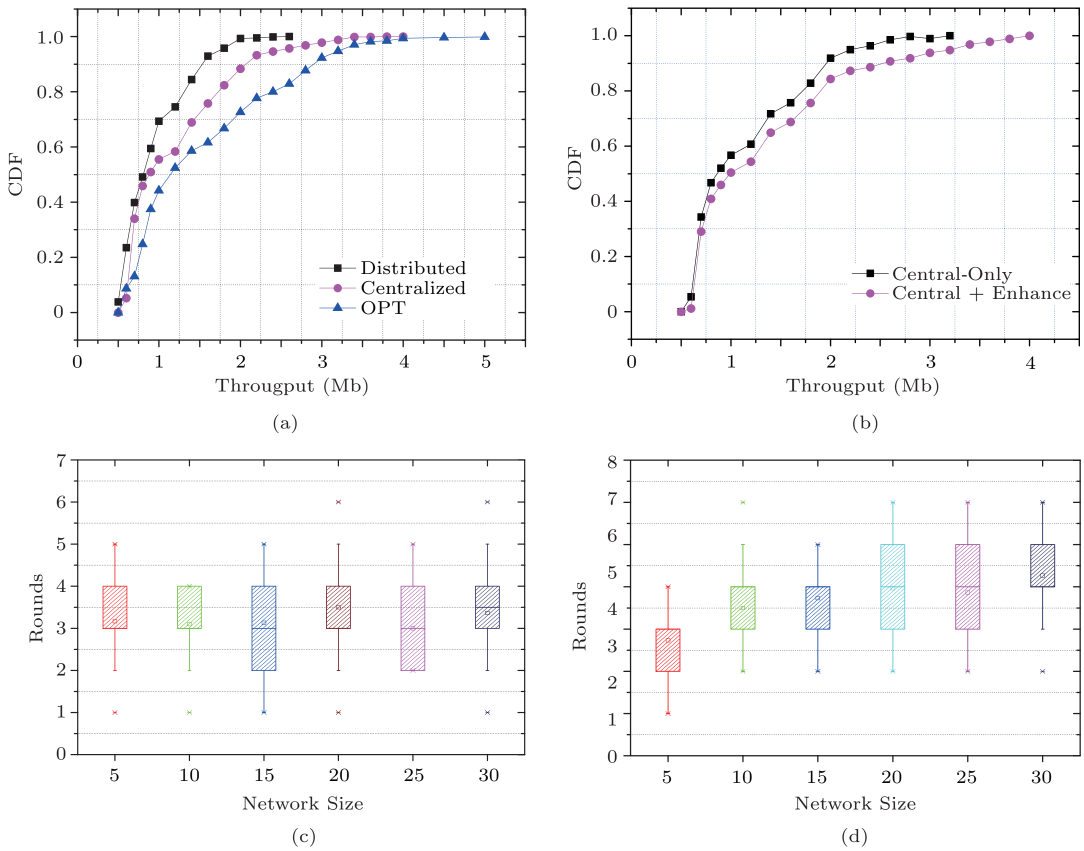

Fig. 4. Performance of enhancement and distributed solution. (a) Distributed solution vs centralized and optimal solution. (b) Centralized solution without enhancement (Central-Only) vs centralized solution with enhancement (Central+Enhance). (c) Cost of distributed solution. (d) Cost of heuristic enhancement.

7 Related Work

Optimal conflict-free channel assignment satisfying a global objective is often NP-hard[5]. This problem can be described as an interference-graph vertex-coloring or edge-coloring problem. Compared with the traditional wireless network, channel assignment in a CRN has to deal with different scopes of spectrum availability. Thus, various distributed approximations were proposed, which are based on observing local interference patterns[23], local bargaining[24], or on coordinations between CR nodes that aim at maximizing some system utility[25-26]. Most recently, the channel assignment problems in a CRN are studied from its dynamic nature. In [6], Yuan *et al.* proposed a time-spectrum model of the available band. Based on it, a set of distributed assignment algorithms were developed. In [27], Gai *et al.* assumed the spectrum opportunity is unknown and modeled it as an arbitrarily-distributed random variable with bounded support, but unknown mean. Under this model, the assignment problem is formulated as a combinatorial multi-armed bandit problem. Different from these studies, our target problem is on the global optimization target of throughput under the SINR interference model, and thus is much more complex.

The SINR model is widely regarded as a better model for interference characterization. Although such a model is preferred, there are many difficulties in carrying out an analysis with this model, due to the computational complexities SINR involves. As a result, there are many previous efforts on single-hop networks, e.g., [28-29]. For multi-hop networks, some efforts[30-32] study cross-layer problems involving multiple layers, aiming at optimal resource allocation or routing. For example, in [31], Bhatia and Kodialam optimized power control and routing, but assumed some frequency hopping mechanism is in place for scheduling, which helps simplify the joint consideration of scheduling. For cross-layer optimization in the SINR model involving three layers (physical, link, and network), nearly all existing efforts (e.g., [33]) followed a layered approach to simplify the analysis. Under such an approach, the solution is obtained by determining the algorithm/mechanism one layer at a time, and then piecing them up together instead of solving a joint optimization problem. These approaches are heuristic at best, and cannot offer any performance guarantee. Compared with these studies, especially [8], the advance of our work could be summarized as follows. Firstly, we work on a more complex problem, considering the physically layer features, e.g., spectrum diversity and NC-OFDM. Secondly, we propose the efficient and accuracy controlled method to improve the accuracy of SINR measurement and estimation, which could greatly improve the optimization results. Thirdly, we provide a heuristic-based enhancement algorithm, which could help to balance the trade-off between the efficiency and the accuracy. Finally, we propose a fully distributed and localized algorithm which could efficiently solve the allocation problem.

8 Conclusions

We studied the throughput optimization problems via spectrum allocation in CR-based WMNs under the physical interference model, which is calibrated by measurements. The complexities brought by CRs, e.g., the NC-OFDM feature, which allows allocating multiple channels into one link, and the spectrum diversity feature, which requires more accurate SINR estimation, are considered. Consequently, we solved this problem in a cross-layer way. In the lower layer, an efficient RSS measurement and an estimation process based on compressive sensing were proposed to replace the propagation gain model. In the upper layer, the optimization problem is modeled as a mixed integer non-linear programming. Both the centralized and the distributed solutions were proposed to solve it, approximately. The centralized solution provides a $(1-\varepsilon)$-approximate result, while the distributed algorithm manages to solve this problem locally and efficiently. Finally, a comprehensive statistical evaluation based on real trace was conducted, whose results illustrate the effectiveness of our proposed solutions.

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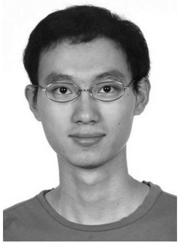

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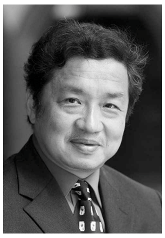

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