Gao QS, Gao XY, Hu Y. A new fuzzy set theory satisfying all classical set formulas. JOURNAL OF COMPUTER SCIENCE AND TECHNOLOGY 24(4): 798–804 July 2009

A New Fuzzy Set Theory Satisfying All Classical Set Formulas

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Received March 24, 2005; revised May 7, 2009.

Abstract A new fuzzy set theory, C-fuzzy set theory, is introduced in this paper. It is a particular case of the classical set theory and satisfies all formulas of the classical set theory. To add a limitation to C-fuzzy set system, in which all fuzzy sets must be "non-uniform inclusive" to each other, then it forms a family of sub-systems, the Z-fuzzy set family. It can be proved that the Z_0-fuzzy set system, one of Z-fuzzy set systems, is equivalent to Zadeh's fuzzy set system. Analysis shows that 1) Zadeh's fuzzy set system defines the relations $A = B$ and $A \subseteq B$ between two fuzzy sets A and B as "$\forall u \in U$, $(\mu_A \in (u) = \mu_B(u))$" and "$\forall u \in U$, $(\mu_A(u) \leqslant \mu_B(u))$" respectively is inappropriate, because it makes all fuzzy sets be "non-uniformly inclusive"; 2) it is also inappropriate to define two fuzzy sets' union and intersection operations as the max and min of their grades of membership, because this prevents fuzzy set's ability to correctly reflect different kinds of fuzzy phenomenon in the natural world. Then it has to work around the problem by invent unnatural functions that are hard to understand, such as augmenting max and min for union and intersection to $\min\{a + b, 1\}$ and $\max\{a + b - 1, 0\}$, but these functions are incorrect on inclusive case. If both pairs of definitions are used together, not only are they unnatural, but also they are still unable to cover all possible set relationships in the natural world; and 3) it is incorrect to define the set complement as $1 - \mu_A(u)$, because it can be proved that set complement cannot exist in Zadeh's fuzzy set, and it causes confusion in logic and thinking. And it is seriously mistaken to believe that logics of fuzzy sets necessarily go against classical and normal thinking, logic, and conception. The C-fuzzy set theory proposed in this paper overcomes all of the above errors and shortcomings, and more reasonably reflects fuzzy phenomenon in the natural world. It satisfies all relations, formulas, and operations of the classical set theory. It is consistent with normal, natural, and classical thinking, logic, and concepts.

Keywords fuzzy set, Zadeh's fuzzy set, classical set, C-fuzzy set

1 Introduction

1.1 Zadeh's Fuzzy Set Theory

A set system is a composition of sets, set relations, set operators, and set operation formulas.

Zadeh's fuzzy set theory[1,2] may be described as follows: assume U is a classical set, called Universe, whose generic elements are denoted as u. A fuzzy subset, A, is defined as $\{(u, \mu_A(u)) | u \in U\}$, where $\mu_A(u)$ is the grade of membership of u in A. $\mu_A(u)$ is a real number satisfying $0 \leqslant \mu_A(u) \leqslant 1$, i.e., $\mu_A(u) \in [0, 1]$, where $[0, 1]$ is a closed real interval. The relations between fuzzy sets A and B, $A = B$ (set equality) and $A \subseteq B$ (A is a subset of B, B includes A) are defined as $\forall u \in U$, $(\mu_A(u) = \mu_B(u))$ and $\forall u \in U$, $(\mu_A(u) \leqslant \mu_B(u))$, respectively. The operations of fuzzy sets, union ($\cup$), intersection ($\cap$), and complement ($\neg$) are defined as $\forall u \in U$, $(\mu_{A\cup B}(u) = \max\{\mu_A(u), \mu_B(u)\})$, $\forall u \in U$, $(\mu_{A\cap B}(u) = \min\{\mu_A(u), \mu_B(u)\})$, and $\forall u \in U$, $(\mu_{\neg A}(u) = 1 - \mu_A(u))$, respectively. Zadeh's fuzzy set theory could not satisfy all formulas of the classical set theory, in particular, when $A \cup \neg A =$ Universe and $A \cap \neg A = \varnothing$ are not true. Zadeh's seems to be an extension to the classical set theory. If we limit the valuation set to $\{0, 1\}$, and define $\mu_A(u) = 1$ when $u \in A$ and $\mu_A(u) = 0$ otherwise, then fuzzy set theory becomes the classical set theory.

1.2 Shortcomings and Mistakes of Zadeh's Fuzzy Set

Since fuzzy set theory was proposed by Zadeh in 1965, it has been developed in theory and applications in the past 40 years. But it has some problems. For further development of fuzzy set theory and application, these problems should not be sidestepped or adopt unscientific explanations similar to astronomy's

Short Paper

This work is supported by the National Natural Science Foundation of China under Grant No. 60343010, the National Basic Research 973 Program of China under Grant No. 2003 CB317000 and the Foundation of Institute of Computing Technology, Chinese Academy of Sciences under Grant No. 20056510.

"epicycle".

When reflected to the real world, there are also some problems with Zadeh's fuzzy set theory. For example, if the grade of membership of a 17-year-old belonging to the youths set is 0.6, and the grade of that 17-year-old belonging to the juvenile set is 0.4, then by normal logic, the grade of the 17-year-old belonging to the set of adolescents (the union of juvenile and youths sets) should be 1.0, but by Zadeh's theory, it is only 0.6. Where is the other 0.4? Then it has to work around the problem by the aid of functions that are unnatural, artificial, and hard to interpret[2], such as $\min\{\mu_A(u)+\mu_B(u),1\}$ and $\max\{\mu_A(u)+\mu_B(u)-1,0\}$[2]. It may be that, they are the only solution to satisfy $F(a,0)=a$, $G(a,0)=0$, $F(a,1)=1$, $G(a,1)=a$, $F(a,1-a)=1$, $G(a,1-a)=0$, and $0\leqslant a,b,F(a,b),G(a,b)\leqslant 1$.

However, for the set inclusion case (A includes B or B includes A), they are incorrect. For example, suppose the grade of membership of 30-year-old in the youth set is 0.2, and in the adolescents set is 0.2, then in the union set of youth and adolescents it should still be 0.2, but it gives 0.4.

If both pairs of set union and intersection formulas are used, then not only do the concepts of union and intersection become hard to explain, they still cannot depict all types of fuzzy phenomenon of the natural world. For instance, see Table 1, let the height of 1.7m belonging to fuzzy sets A (tall), B (med), C (small) and D (short) be 0.1, 0.7, 0.1 and 0.1. Then the grade of E (med-tall) $=A\cup B$ and F (med-small) $=B\cup C$ are both 0.8. And, the grade of intersection $(E\cap F)$ and union $(E\cup F)$ should be $\mu_{E\cap F}(1.7)=0.7$ and $\mu_{E\cup F}(1.7)=0.9$. But by Zadeh's they are $\mu_{E\cap F}(1.7)=\min\{0.8,0.8\}=0.8$ and $\mu_{E\cup F}(1.7)=\max\{0.8,0.8\}=0.8$, or by both $\mu_{E\cap F}(1.7)=\max\{0.8+0.8-1,0\}=0.6$ and $\mu_{E\cup F}(1.7)=\min\{0.8+0.8,1\}=1$. None of them are correct!

Let $H=A\cup B\cup C$, $\forall u\in U$, $(\mu_H(u)=1-\mu_C(u)\})$ be true, where H includes C, but is not C's complement.

Table 1. Examples

u	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90 and over
A	0.0	0.0	0.0	0.1	0.3	0.7	0.9	1.0
B	0.0	0.2	0.6	0.7	0.7	0.3	0.1	0.0
C	0.5	0.4	0.2	0.1	0.0	0.0	0.0	0.0
D	0.5	0.4	0.2	0.1	0.0	0.0	0.0	0.0

The fact is that set complement cannot exist in Zadeh's fuzzy set theory (see Theorem 6). Zadeh made a mistake in defining a complement set. It causes $A\cup\neg A=$ Universe and $A\cap\neg A=\varnothing$ to be not true. It means that there exist elements that belong to neither A nor $\neg A$, and elements that belong to both A and $\neg A$. If $\forall u\in U$, $(\mu_A(u)=0.5)$, then it means that $A=\neg A$. (Remark: Zadeh's fuzzy set theory mistakenly define conjugate as complement. "$A=\neg A$" is self-conjugate.)

This is beyond "hard to interpret"[2]. It causes confusion in logic and thinking. Moreover, it causes seriously mistaken belief that logic of fuzzy sets would necessarily go against classical and normal thinking, logic, and concepts.

Why does Zadeh's fuzzy set theory have these problems? Can fuzzy set theory overcome these problems? Can fuzzy set theory satisfy all formulas of classical set theory? Can fuzzy set be a particular case of the classical set theory? What do $\min\{\mu_A(u),\mu_B(u)\}$, $\max\{\mu_A(u),\mu_B(u)\}$, $\min\{\mu_A(u)+\mu_B(u),1\}$, and $\max\{\mu_A(u)+\mu_B(u)-1,0\}$ represent? The following subsections will give answers and constructive proofs to these questions.

1.3 Some Other Papers about Fuzzy Set

M. Shimoda[3] presented a new and natural interpretation of fuzzy sets and fuzzy relations, but still did not change the fact that it could not satisfy all formulas of the classical set system.

G. Coletti and R. Scozzafava[4] presented an opinion that fuzzy sets are corresponding to conditional probability. For example, $E=\{$Mary is young$\}$, $A_x=\{$the age of Mary is $x\}$, $H_0=\{$Mary's age is greater than 40$\}$, and $H_1=\{$Mary's age is less than 25$\}$ are events. The grade of membership of u in fuzzy set E, $\mu_E(u)$, is corresponding to conditional probability $P(E|A_x)$, and $\mu_E(x>40)=0$, $\mu_E(x<25)=1$ are corresponding to conditional probability $P(E|H_0)=0$, $P(E|H_1)=1$ separately. But in this paper, nothing about shortcomings and mistakes of Zadeh's fuzzy set theory were discussed, and corresponding relation discussed is also nonessential.

A. Piegat[5] presented a new definition of the fuzzy set: a fuzzy set A of the elements x is a collection of the elements $\{x|x\in X\}$, which possess a specific property pA of the set and are qualified in the set by a qualifier QA using a qualification algorithm $Q\mathrm{Alg}_A$. But nothing about essential shortcomings and mistakes of Zadeh's fuzzy set theory and how to overcome them completely was discussed in it.

2 New Fuzzy Set Theory: *C*-Fuzzy Set Theory

2.1 *C*-Fuzzy Set Theory

Definition 1. *Assume U and V are classical sets. Let u be a generic element of U, v be a generic element of V, where V is the set of all real numbers in a closed real interval $[0,1]$. Let $U'=U\times V$, and (u,v) be a generic element of U'. A fuzzy set A in C-fuzzy set* *theory can be expressed as* $\{(u, Y_A(u), \mu_A(u))|Y_A(u) \subseteq V,\ u \in U,\ \mu_A(u) = |Y_A(u)| \in [0,1]\}$, *where* $|X|$ *is the measure of* X *and* $[a,b]$ *denotes the set of real numbers in a closed interval* $[a,b]$.

Remark. 1) If V is the set of all real numbers in a closed real interval $[a,b]$, then the measure is the Lebesgue measure[5]. And assume that X is empty if and only if $|X| = 0$, and that X is universe if and only if $|X| = 1$.

2) If V is a finite set of N elements, then define measure of each element as $1/N$ or $|v_i|$, where $\sum_{i=1}^{N} |v_i| = 1$.

Definition 2. *Assume* U, V_1 *and* V_2 *are classical sets. Let* u *be a generic element of* U, v_i *be a generic element of* V_i, V_i *is the set of all real numbers in a closed interval* $[0,1]$. *Let* $U'_2 = U \times V_1 \times V_2$, *and* (u, v_1, v_2) *be a generic element of* U'_2. *A fuzzy set* A *in* C_2*-fuzzy set theory can be expressed as* $\{(u, Z_A(u), \mu_A(u))|Z_A(u) = \{(v_1, Y_{2A}(u, v_1))|v_1 \in Y_{1A}(u) \subseteq V_1,\ Y_{2A}(u, v_1) \subseteq V_2\},\ \mu_A(u) = |Z_A(u)| \in [0,1],\ u \in U\}$, *where* $|X|$ *is the measure of* X.

Remark. This definition can be further extended to C_k-Fuzzy Set theory, where $k \geqslant 2$.

Below we will focus on C-fuzzy set theories. C_k-fuzzy set theories are similar, thus omitted, where $k \geqslant 2$.

Definition 3. *The set inclusion* $(A \subseteq B)$ *and equality* $(A = B)$ *relationships of arbitrary two* C*-fuzzy sets,* A *and* B, *are defined as:* $\forall u \in U$, $(Y_A(u) \subseteq Y_B(u))$ *and* $\forall u \in U$, $(Y_A(u) = Y_B(u))$ *respectively.*

Definition 4. *The set union* $(A \cup B)$, *intersection* $(A \cap B)$, *and complement* $(\neg A)$ *operations of arbitrary two* C*-fuzzy sets,* A *and* B, *are defined respectively as*:

1) $A \cup B = \{(u, Y_{A\cup B}(u),\ \mu_{A\cup B}(u))|Y_{A\cup B}(u) = Y_A(u) \cup Y_B(u) \subseteq [0,1],\ \mu_{A\cup B}(u) = |Y_{A\cup B}(u)| \in [0,1],\ u \in U\}$;

2) $A \cap B = \{(u, Y_{A\cap B}(u),\ \mu_{A\cap B}(u))|Y_{A\cap B}(u) = Y_A(u) \cap Y_B(u) \subseteq [0,1],\ \mu_{A\cap B}(u) = |Y_{A\cap B}(u)| \subseteq [0,1],\ u \in U\}$; *and*

3) $\neg A = \{(u, Y_{\neg A}(u),\ \mu_{\neg A}(u))|Y_{\neg A}(u) = \neg Y_A(u) \subseteq [0,1],\ \mu_{\neg A}(u) = |Y_{\neg A}(u)| \in [0,1],\ u \in U\}$.

Theorem 1. *In the* C*-fuzzy set theory* 1) *all classical set relations and operators exist and all formulas still hold, including* $A \cup \neg A = Universe$ *and* $A \cap \neg A = \varnothing$.

2) *any two fuzzy sets* $A = \{(u, Y_A(u), \mu_A(u))|Y_A(u) \subseteq [0,1],\ u \in U,\ \mu_A(u) = |Y_A(u)| \in [0,1],\ u \in U\}$ *and* $B = \{(u, Y_B(u),\ \mu_B(u))|Y_B(u) \subseteq [0,1],\ \mu_B(u) = |Y_B(u)| \in [0,1],\ u \in U\}$ *satisfy*

2.1) $A = B$ *if and only if* $\forall u \in U$, $(Y_A(u) = Y_B(u))$;

2.2) $A \subseteq B$ *if and only if* $\forall u \in U$, $(Y_A(u) \subseteq Y_B(u))$, *and similarly for* $\subset$, $\supseteq$, *and* $\supset$;

2.3) $\forall u \in U$, $(Y_{A\cap B}(u) = Y_A(u) \cap Y_B(u))$;

2.4) $\forall u \in U$, $(Y_{A\cup B}(u) = Y_A(u) \cup Y_B(u))$;

2.5) $\forall u \in U$, $(Y_{\neg A}(u) = \neg Y_A(u))$.

3) *if* $A = F(A_1, A_2, \ldots, A_N)$ *is a finite function performing a combination of* $\cup$, $\cap$, *and* $\neg$ *operations, then* $\forall u \in U$, $(Y_{F(A_1,A_2,\ldots,A_N)}(u) = F(Y_{A_1}(u), Y_{A_2}(u), \ldots, Y_{A_N}(u)))$ *where,* $A_i = \{(u, Y_{A_i}(u),\ \mu_{A_i}(u))|Y_{A_i}(u) \subseteq [0,1],\ \mu_{A_i}(u) = |Y_{A_i}(u)| \in [0,1],\ u \in U\}$, $i = 1, 2, \ldots, N$, *are arbitrary* C*-fuzzy sets.*

Proof. By the fact that $\mu_X(u) = |Y_X(u)|$.

In fact, C-fuzzy set $\{(u, Y_A(u), \mu_A(u))|Y_A(u) \subseteq [0,1],\ u \in U,\ \mu_A(u) = |Y_A(u)| \in [0,1]\}$ can be simply written as $\{(u, Y_A(u),\ |Y_A(u)|)|Y_A(u) \subseteq [0,1],\ u \in U\}$, and further simplified as $\{(u, Y_A(u))|Y_A(u) \subseteq [0,1],\ u \in U\}$. Obviously, this is classical set theory. Of course, it contains all classical set relations and operators, moreover, it satisfies all classical set formulas. □

2.2 Calculation of Grade of Membership in the *C*-Fuzzy Set Theory

2.2.1 *Partition of* $V = [0,1]$

For arbitrary new fuzzy sets $A = \{(u, Y_A(u), \mu_A(u))|Y_A(u) \subseteq [0,1], \mu_A(u) = |Y_A(u)| \in [0,1], u \in U\}$ and $B = \{(u, Y_B(u),\ \mu_B(u))|Y_B(u) \subseteq [0,1],\ \mu_B(u) = |Y_B(u)| \in [0,1],\ u \in U\}$, for arbitrary $u \in U$, sets $Y_A(u)$ and $Y_B(u)$ partitions $V = [0,1]$ into 4 parts: $Y_A(u) \cap Y_B(u)$, $Y_A(u) \cap \neg Y_B(u)$, $\neg Y_A(u) \cap Y_B(u)$, and $\neg Y_A(u) \cap \neg Y_B(u)$.

Lemma 1. For any new fuzzy sets $A = \{(u, Y_A(u), \mu_A(u))|Y_A(u) \subseteq [0,1], \mu_A(u) = |Y_A(u)| \in [0,1], u \in U\}$ and $B = \{(u, Y_B(u),\ \mu_B(u))|Y_B(u) \subseteq [0,1],\ \mu_B(u) = |Y_B(u)| \in [0,1], u \in U\}$, it is true that

1) $\forall u \in U$, $(\mu_A(u) = \mu_{A\cap B}(u) + \mu_{A\cap\neg B}(u))$ and $\forall u \in U$, $(\mu_B(u) = \mu_{A\cap B}(u) + \mu_{\neg A\cap B}(u))$;

2) $\forall u \in U$, $(\mu_{A\cap B}(u) + \mu_{A\cap\neg B}(u) + \mu_{\neg A\cap B}(u) + \mu_{\neg A\cap\neg B}(u) = 1)$;

3) $\forall u \in U$, $(\mu_{A\cup B}(u) = \mu_{A\cap\neg B}(u) + \mu_{\neg A\cup B}(u) + \mu_{A\cap B}(u) = \mu_A(u) + \mu_B(u) - \mu_{A\cap B}(u))$;

4) $\forall u \in U$, $(\mu_{A\cup B}(u) + \mu_{A\cap B}(u) = \mu_A(u) + \mu_B(u))$.

2.2.2 *Calculation of Grade of Membership of* $\neg A$

Theorem 2. *For any new fuzzy set* $A = \{(u, Y_A(u), \mu_A(u))|Y_A(u) \subseteq [0,1],\ \mu_A(u) = |Y_A(u)| \in [0,1],\ u \in U\}$, *it is true that* $\forall u \in U$, $(\mu_{\neg A}(u) = 1 - \mu_A(u))$.

Proof. From Definition 4, Theorem 1 and Lemma 1, we have $\forall u \in U$, $(\mu_A(u) + \mu_{\neg A}(u) = |Y_A(u)| + |Y_{\neg A}(u)| = |Y_A(u)| + |\neg Y_A(u)| = |[0,1]| = 1)$. □

2.2.3 *Calculation of Grade of Membership of* $A \cap B$ *and* $A \cup B$

Definition 5. *Define "A and B are inclusive" as* $A \subseteq B$ *or* $B \subseteq A$ *(i.e., either* B *includes* A *or* A

includes B), where A and B are sets.

Definition 6. *Define "A and B are non-uniformly inclusive" as for all* $u \in U$, $Y_A(u)$ *and* $Y_B(u)$ *are inclusive, where A and B are sets,* $Y_A(u) \subseteq [0,1]$, *and* $Y_B(u) \subseteq [0,1]$.

Theorem 3. *For arbitrary C-fuzzy sets* $A = \{(u, Y_A(u), \mu_A(u)) | Y_A(u) \subseteq [0,1],\ \mu_A(u) = |Y_A(u)| \in [0,1],\ u \in U\}$ *and* $B = \{(u, Y_B(u), \mu_B(u)) | Y_B(u) \in [0,1],\ \mu_B(u) = |Y_B(u)| \in [0,1],\ u \in U\}$, *it is true for any* $u \in U$, *that*

1) $Y_A(u)$ *not intersect* $Y_B(u)$ *if and only if* $\mu_{A\cap B}(u) = 0$, *if and only if* $\mu_{A\cup B}(u) = \mu_A(u) + \mu_B(u)$;

2) $Y_A(u)$ *and* $Y_B(u)$ *are inclusive if and only if* $\mu_{A\cap B}(u) = \min\{\mu_A(u),\ \mu_B(u)\}$, *if and only if* $\mu_{A\cup B}(u) = \max\{\mu_A(u), \mu_B(u)\}$;

3) $Y_A(u)$ *and* $Y_B(u)$ *are not inclusive if and only if* $\mu_{A\cap B}(u) < \min\{\mu_A(u), \mu_B(u)\}$ *if and only if* $\mu_{A\cup B}(u) > \max\{\mu_A(u), \mu_B(u)\}$. (*See Tables* 2 *and* 3.)

Proof. By Definition 1, Definition 3, Definition 4, and Lemma 1. □

Corollary 1. *For arbitrary C-fuzzy sets* $A = \{(u, Y_A(u), \mu_A(u)) | Y_A(u) \subseteq [0,1],\ \mu_A(u) = |Y_A(u)| \in [0,1],\ u \in U\}$ *and* $B = \{(u, Y_B(u), \mu_B(u)) | Y_B(u) \subseteq [0,1],\ \mu_B(u) = |Y_B(u)| \in [0,1],\ u \in U\}$,

1) *A does not intersect B if and only if* $\forall u \in U$, $(\mu_{A\cap B}(u) = 0)$, *if and only if* $\forall u \in U$, $(\mu_{A\cup B}(u) = \mu_A(u) + \mu_B(u))$.

2) $Y_A(u)$ *and* $Y_B(u)$ *are non-uniformly inclusive if and only if* $\forall u \in U$, $(\mu_{A\cap B}(u) = \min\{\mu_A(u), \mu_B(u)\})$, *if and only if* $\forall u \in U$, $(\mu_{A\cup B}(u) = \max\{\mu_A(u), \mu_B(u)\})$.

Remark. If A and B are inclusive then A and B are non-uniformly inclusive, but the reverse if not necessarily true: if A and B are non-uniformly inclusive, A and B may not be inclusive.

2.3 Correlation Coefficient $\zeta_{B/A}(u)$ and Unified Calculation of $\mu_A(u)$ in the C-Fuzzy Set

Definition 7. *For arbitrary C-fuzzy set* $A = \{(u, Y_A(u), \mu_A(u)) | Y_A(u) \subseteq [0,1],\ \mu_A(u) = |Y_A(u)| \in [0,1],\ u \in U\}$ *and* $B = \{(u, Y_B(u), \mu_B(u)) | Y_B(u) \subseteq [0,1],\ \mu_B(u) = |Y_B(u)| \in [0,1],\ u \in U\}$, *call* $\zeta_{B/A}(u) = \mu_{A\cap B}(u)/\mu_A(u)$ *the correlation coefficient of A's grade of membership on B's.*

Obviously, $\zeta_{B/A}(u) + \zeta_{\neg B/A}(u) = 1$.

Theorem 4. *For arbitrary C-fuzzy set* $A = \{(u, Y_A(u), \mu_A(u)) | Y_A(u) \subseteq [0,1],\ \mu_A(u) = |Y_A(u)| \in [0,1],\ u \in U\}$ *and* $B = \{(u, Y_B(u), \mu_B(u)) | Y_B(u) \subseteq [0,1],\ \mu_B(u) = |Y_B(u)| \in [0,1],\ u \in U\}$, *the following are true:*

1) $\forall u \in U$, $(\mu_{A\cap B}(u) = \mu_A(u) - \mu_{A\cap\neg B}(u) = \mu_B(u) - \mu_{\neg A\cap B}(u) = \mu_A(u) - \mu_A(u) \times \zeta_{\neg B/A}(u) = \mu_B(u) - \mu_B(u) \times \zeta_{\neg A/B}(u))$;

2) $\forall u \in U$, $(\mu_{A\cup B}(u) = \mu_A(u) + \mu_{\neg A\cap B}(u) = \mu_B(u) + \mu_{A\cap\neg B}(u) = \mu_A(u) + \mu_B(u) \times \zeta_{\neg A/B}(u) = \mu_B(u) + \mu_A(u) \times \zeta_{\neg B/A}(u))$.

Proof. By Definition 7 and Lemma 1. □

Theorem 4 gives unified calculations for fuzzy set operations in the C-fuzzy set theory (see Table 4). For convenience of calculation, if A and B do not intersect or are not inclusive, simpler equations (see Table 5) can be used, although they are equivalent. Table 6 describes the relationship between $\zeta(u)$ and the sets.

Table 2. Calculation of Grade of Membership in C-Fuzzy Set Theories (for any u, $u \in U$)

	$Y_A(u)$ and $Y_B(u)$ are not intersect	$Y_A(u)$ and $Y_B(u)$ are inclusive	$Y_A(u)$ and $Y_B(u)$ are not inclusive
$A \cap B$	$\mu_{A\cap B}(u) = 0$	$\mu_{A\cap B}(u) = \min(\mu_A(u), \mu_B(u))$	$\mu_{A\cap B}(u) < \min(\mu_A(u), \mu_B(u))$
$A \cup B$	$\mu_{A\cup B}(u) = \mu_A(u) + \mu_B(u)$	$\mu_{A\cup B}(u) = \max(\mu_A(u), \mu_B(u))$	$\mu_{A\cup B}(u) > \max(\mu_A(u), \mu_B(u))$
Necessary and sufficient condition	$\mu_{A\cap B}(u) = 0$ or $\mu_{A\cup B}(u) = \mu_A(u) + \mu_B(u)$	$\mu_{A\cap B}(u) = \min(\mu_A(u), \mu_B(u))$, or $\mu_{A\cup B}(u) = \max(\mu_A(u), \mu_B(u))$	$\mu_{A\cap B}(u) < \min(\mu_A(u), \mu_B(u))$ or $\mu_{A\cup B}(u) > \max(\mu_A(u), \mu_B(u))$

Table 3. Calculation of Grade of Membership in C-Fuzzy Set Theories (Cont.)

	A and B are not intersect	A and B are non-uniformly inclusive
$A \cap B$	$\forall u \in U$, $(\mu_{A\cap B}(u) = 0)$	$\forall u \in U$, $(\mu_{A\cap B}(u) = \min(\mu_A(u), \mu_B(u)))$
$A \cup B$	$\forall u \in U$, $(\mu_{A\cup B}(u) = \mu_A(u) + \mu_B(u))$	$\forall u \in U$, $(\mu_{A\cup B}(u) = \max(\mu_A(u), \mu_B(u)))$
Necessary and sufficient condition	$\forall u \in U$, $(\mu_{A\cap B}(u) = 0)$ or $\mu_{A\cup B}(u) = \mu_A(u) + \mu_B(u)$	$\forall u \in U$, $(\mu_{A\cap B}(u) = \min(\mu_A(u), \mu_B(u)))$ or $\forall u \in U$, $(\mu_{A\cup B}(u) = \max(\mu_A(u), \mu_B(u)))$

Table 4. Unified Calculation of Grade of Membership in the C-Fuzzy Set ($\forall u \in U$)

$A \cap B$	$A \cup B$	$\neg A$
$\mu_{A\cap B}(u) = \mu_A(u) - \mu_A(u) \times \zeta_{\neg B/A}(u)$	$\mu_{A\cup B}(u) = \mu_A(u) + \mu_B(u) \times \zeta_{\neg A/B}(u)$	$\mu_{\neg A}(u) = 1 - \mu_A(u)$

Table 5. Unified Calculation of Grade of Membership and Dependence in the C-Fuzzy Set (Cont.)

	$A \cap B$	$A \cup B$
$Y_A(u)$ and $Y_B(u)$ do not intersect	$\mu_{A\cap B}(u) = \mu_A(u) - \mu_A(u) \times \zeta_{\neg B/A}(u)$ $= \mu_B(u) - \mu_B(u) \times \zeta_{\neg A/B}(u) = 0$	$\mu_{A\cup B}(u) = \mu_A(u) + \mu_B(u) \times \zeta_{\neg A/B}(u) = \mu_B(u) + \mu_A(u) \times \zeta_{\neg B/A}(u) = \mu_A(u) + \mu_B(u)$
$Y_A(u)$ and $Y_B(u)$ are inclusive	$\mu_{A\cap B}(u) = \mu_A(u) - \mu_A(u) \times \zeta_{\neg B/A}(u)$ $= \mu_B(u) - \mu_B(u) \times \zeta_{\neg A/B}(u)$ $= \min(\mu_A(u), \mu_B(u))$	$\mu_{A\cup B}(u) = \mu_A(u) + \mu_B(u) \times \zeta_{\neg A/B}(u)$ $= \mu_B(u) + \mu_A(u) \times \zeta_{\neg B/A}(u)$ $= \max(\mu_A(u), \mu_B(u))$
$Y_A(u)$ intersect $Y_B(u)$ but not inclusive	$\mu_{A\cap B}(u) = \mu_A(u) - \mu_A(u) \times \zeta_{\neg B/A}(u)$ $= \mu_B(u) - \mu_B(u) \times \zeta_{\neg A/B}(u)$	$\mu_{A\cup B}(u) = \mu_A(u) + \mu_B(u) \times \zeta_{\neg A/B}(u)$ $= \mu_B(u) + \mu_A(u) \times \zeta_{\neg B/A}(u)$

Table 6. Relationship Between Correlation Coefficient $\zeta_{A/B}(u)$ and Relation of the Sets

$Y_A(u)$ and $Y_B(u)$ are not intersect	$\zeta_{A/B}(u) = 0$	$\zeta_{B/A}(u) = 0$
$Y_A(u) = Y_B(u)$	$\zeta_{A/B}(u) = 1$	$\zeta_{B/A}(u) = 1$
$Y_A(u) \subseteq Y_B(u)$	$\zeta_{A/B}(u) = \mu_A(u)/\mu_B(u)$	$\zeta_{B/A}(u) = 1$
$Y_A(u)$ intersect $Y_B(u)$ but not inclusive	$\zeta_{A/B}(u) = \mu_{A\cap B}(u)/\mu_B(u)$	$\zeta_{B/A}(u) = \mu_{A\cap B}(u)/\mu_A(u)$

3 A New Definition for Zadeh's Fuzzy Set Theory — A Particular Case of C-Fuzzy Set Theory

3.1 Z_0-Fuzzy Set System in Z-Fuzzy Set System Family — A Particular Case of C-Fuzzy Set Theory

Definition 8. *A Z-fuzzy set system is a group of C-fuzzy sets that satisfy the T-condition, which states for each $u \in U$, all $Y_X(u)$ are inclusive of each other, where X is an arbitrary fuzzy set in this Z-fuzzy set system, $X = \{(u, Y_X(u),\ \mu_X(u)) | Y_X(u) \subseteq [0,1],\ \mu_X(u) = |Y_X(u)| \in [0,1],\ u \in U\}$.*

There are infinite Z-fuzzy set systems, including the following: $Y_A(u) = [e(1-\mu_A(u)),\ \mu_A(u)+e(1-\mu_A(u))]$, where $0 \leqslant e \leqslant 1$. The simplest of those is the Z_0-fuzzy set system, which is unique.

Definition 9. *Assume U is a classical set, called Universe, whose generic elements are denoted as u. A Z_0-fuzzy set, A, is defined as $\{(u, [0, \mu_A(u)], \mu_A(u)) | u \in U,\ [0, \mu_A(u)] \subseteq [0,1],\ \mu_A(u) \in [0,1], \}$, where $\mu_A(u)$ is called grade of membership of u in A, $\mu_A(u)$ is a real number satisfying $0 \leqslant \mu_A(u) \leqslant 1$. $[0, \mu_A(u)]$ is a subset of classical set $[0,1]$. That is, the Z_0-fuzzy set system is one of the Z-fuzzy set system family, where every fuzzy set $A = \{(u, Y_A(u), \mu_A(u)) | Y_A(u) \subseteq [0,1],\ \mu_A(u) = |Y_A(u)| \in [0,1],\ u \in U\}$ satisfies $Y_A(u) = [0, \mu_A(u)]$.*

It is obvious that Z_0-fuzzy set, Z-fuzzy set, and C-fuzzy set all are the classical set.

Theorem 5. *In a sub-system of the C-fuzzy set system, the following are mutually necessary and sufficient*:

1) *any two fuzzy sets of this sub-system, A and B, satisfy $\forall u \in U$, $(\mu_A(u) \leqslant \mu_B(u))$ if and only if $A \subseteq B$*;

2) *any two fuzzy sets of this sub-system, A and B, satisfy A and B are non-uniformly inclusive*;

3) *any two fuzzy sets of this sub-system, A and B, satisfy $\forall u \in U$, $(\mu_D(u) = \mu_{A\cap B}(u) = \min\{\mu_A(u), \mu_B(u)\})$;*

4) *any two fuzzy sets of this sub-system, A and B, satisfy $\forall u \in U$, $(\mu_D(u) = \mu_{A\cup B}(u) = \max\{\mu_A(u), \mu_B(u)\})$;*

5) *this fuzzy set system is a Z-fuzzy set system.*

Remark. Note that:

1) For any $u \in U$, $(\mu_A(u) \leqslant \mu_B(u)) \vee (\mu_B(u) \leqslant \mu_A(u))$ is true, so if 1) is true, then $Y_A(u) \subseteq Y_B(u) \vee Y_B(u) \subseteq Y_A(u)$ is true, therefore A and B are non-uniformly inclusive 2);

2) By 2) of Theorem 3, "$Y_A(u)$ and $Y_B(u)$ are inclusive if and only if $\mu_{A\cap B}(u) = \min\{\mu_A(u),\ \mu_B(u)\}$ if and only if $\mu_{A\cup B}(u) = \max\{\mu_A(u),\ \mu_B(u)\}$".

Definition 10. *If Z_0-fuzzy sets A and B satisfy $\forall u \in U$, $(\mu_A(u) + \mu_B(u) = 1)$, then A and B are conjugates, or A is conjugate with B, or B is conjugate with A, and denoted as $B = \Theta A$, or $A = \Theta B$.*

It is obvious that, in C-fuzzy set system (or Z-fuzzy set system), the fuzzy set B that conjugates with fuzzy set A is not unique. But in Z_0-fuzzy set system, the fuzzy set B that conjugates with fuzzy set A is unique.

Theorem 6. *Fuzzy sets in the Z-fuzzy set system (including Z_0-fuzzy set system), except for the Universe and the Empty Set, do not have set complement ($\neg$) operation.*

Proof. Suppose a Z-fuzzy set A that is non-Universal and non-empty, and it has a complement $\neg A$, then A and $\neg A$ do not satisfy the T-condition. □

Lemma 2. *In Z_0-fuzzy set system, $A = B$ if and only if $\forall u \in U$, $(\mu_A(u) = \mu_B(u))$; $A \subseteq B$ if and only if $\forall u \in U$, $(\mu_A(u) \leqslant \mu_B(u))$; $D = A \cup B$ if and only if $\forall u \in U$, $(\mu_D(u) = \mu_{A\cup B}(u) = \max\{\mu_A(u),\ \mu_B(u)\})$; $D = A \cap B$ if and only if $\forall u \in U$, $(\mu_D(u) = \mu_{A\cap B}(u) = \min\{\mu_A(u),\ \mu_B(u)\})$; and $D = \Theta A$ if and only if $\forall u \in U$, $(\mu_D(u) = \mu_{\Theta A}(u) = 1 - \mu_A(u))$, where ΘA is conjugate with A.*

Proof. It is obvious by Definitions 3, 4, 6, 9, 10,

Table 7. Comparison of C-Fuzzy Set Theory and Zadeh's Fuzzy Set Theory

	Zadeh's Fuzzy Set Theory	C-Fuzzy Set Theory
Grade of Membership	Grade of membership is not a measure. It could be defined as a component of the fuzzy set theory.	Grade of membership is a measure. It could not be defined as a component of the fuzzy set theory.
Unified Calculation of Grade of Membership	Impossible to exist.	Exists, $\mu_{A\cap B}(u)=\mu_A(u)-\mu_A(u)\times\zeta_{\neg B/A}(u)$; $\mu_{A\cup B}(u)=\mu_A(u)+\mu_B(u)\times\zeta_{\neg A/B}(u)$.
A and B Non-Uniformly Inclusive	$\mu_{A\cap B}(u)=\min\{\mu_A(u),\mu_B(u)\}$; $\mu_{A\cup B}(u)=\max\{\mu_A(u),\mu_B(u)\}$.	$\mu_{A\cap B}(u)=\mu_A(u)-\mu_A(u)\times\zeta_{\neg B/A}(u)$ $=\min\{\mu_A(u),\mu_B(u)\}$; $\mu_{A\cup B}(u)=\mu_A(u)+\mu_B(u)\times\zeta_{\neg A/B}(u)$ $=\max\{\mu_A(u),\mu_B(u)\}$.
A and B Non-Intersect	Does not exist, and hard to be interpreted[2]: $\mu_{A\cap B}(u)=\max\{0,\mu_A(u)+\mu_B(u)-1\}$; $\mu_{A\cup B}(u)=\min\{\mu_A(u)+\mu_B(u),1\}$.	Exists, and easy to be understood: $\mu_{A\cap B}(u)=\mu_A(u)-\mu_A(u)\times\zeta_{\neg B/A}(u)=0$; $\mu_{A\cup B}(u)=\mu_A(u)+\mu_B(u)\times\zeta_{\neg A/B}(u)=\mu_A(u)+\mu_B(u)$.
A and B Intersect but Not Non-Uniformly Inclusive	Does not exist, and cannot express.	$\mu_{A\cap B}(u)=\mu_A(u)-\mu_A(u)\times\zeta_{\neg B/A}(u)$; $\mu_{A\cup B}(u)=\mu_A(u)+\mu_B(u)\times\zeta_{\neg A/B}(u)$.
Correctly Depict Natural World, in Both Cases of "A and B Non-Intersect" and "Non-Uniformly Inclusive"	It cannot be expressed. For Youth-Juvenile: incorrect $\mu_{A\cap B}(u)=\min\{\mu_A(u),\mu_B(u)\}$; $\mu_{A\cup B}(u)=\max\{\mu_A(u),\mu_B(u)\}$. For Youth-Adolescent: incorrect $\mu_{A\cap B}(u)=\max\{0,\mu_A(u)+\mu_B(u)-1\}$; $\mu_{A\cup B}(u)=\min\{\mu_A(u)+\mu_B(u),1\}$.	It can be expressed. For Youth-Juvenile: correct $\mu_{A\cap B}(u)=\mu_A(u)-\mu_A(u)\times\zeta_{\neg B/A}(u)=0$; $\mu_{A\cup B}(u)=\mu_A(u)+\mu_B(u)\times\zeta_{\neg A/B}(u)=\mu_A(u)+\mu_B(u)$. For Youth-Adolescent: correct $\mu_{A\cap B}(u)=\mu_A(u)-\mu_A(u)\times\zeta_{\neg B/A}(u)=\min\{\mu_A(u),\mu_B(u)\}$; $\mu_{A\cup B}(u)=\mu_A(u)+\mu_B(u)\times\zeta_{\neg A/B}(u)=\max\{\mu_A(u),\mu_B(u)\}$.
Correctly Depict Natural World, in Case of "A and B intersect but are not non-uniformly inclusive"	It cannot be expressed.	It can be expressed. $\mu_{A\cap B}(u)=\mu_A(u)-\mu_A(u)\times\zeta_{\neg B/A}(u)$; $\mu_{A\cup B}(u)=\mu_A(u)+\mu_B(u)\times\zeta_{\neg A/B}(u)$.
Complement ($\neg$)	Does not exist. Erroneously conjugates to be treated as complement. Satisfy: $\mu_{\neg A}(u)=1-\mu_A(u)$.	Exists unique Complement $\mu_{\neg A}(u)=1-\mu_A(u)$. Contains multiple conjugates. Satisfies: $\mu_{\neg A}(u)=1-\mu_A(u)$.
Satisfy All Classical Set Relations, Operations, and Formulas.	Does not satisfy $A\cup\neg A=$ Universe and $A\cap\neg A=\varnothing$. Exists elements that is not A and not $\neg A$. Logical and conception confusion.	Satisfies. Satisfies $A\cup\neg A=$ Universe and $A\cap\neg A=\varnothing$. Conform to natural logic and concepts.
Conform to Natural Thinking	Cannot. If grade of membership of a in A is 0.4, in $\neg A$ is 0.6, then the grade of membership of a in A and $\neg A$ should be 0, but it is 0.4 instead.	Conform. If grade of membership of a in A is 0.4, in $\neg A$ is 0.6, then the grade of membership of a in A and $\neg A$ is 0.

Lemma 1, and Theorem 1. Note that Z_0-fuzzy set system is a group of C-fuzzy sets with limitation that $\forall u\in U$, $(Y_X(u)=[0,\mu_X(u)])$, where X is an arbitrary Z_0-fuzzy set. □

3.2 Z_0-Fuzzy Set System — Equality to a New Definition of Zadeh's Fuzzy Set Theory

Lemma 3. *Any Z_0-fuzzy set $A=\{(u,Y_A(u),\mu_A(u))|Y_A(u)=[0,\mu_A(u)]\subseteq[0,1],\ \mu_A(u)=|Y_A(u)|\in[0,1],\ u\in U\}$ in the Z_0-fuzzy set system can be written simply as $A=\{(u,[0,\mu_A(u)],\mu_A(u))|\mu_A(u)\in[0,1],\ u\in U\}$, and further simplified as $A=\{(u,[0,\mu_A(u)])|[0,\mu_A(u)]\subseteq[0,1],\ u\in U\}$ or just $A=\{(u,\mu_A(u))|\mu_A(u)\in[0,1],\ u\in U\}$.*

Proof. Just note that for any $u\in U$, $[0,\mu_A(u)]$ and $\mu_A(u)$ uniquely determines each other, $[0,\mu_A(u)]\subseteq[0,1]$ if and only if $\mu_A(u)\in[0,1]$. □

Lemma 4. *Let A and B be Z_0-fuzzy sets in Z_0-fuzzy set system, then $\forall u\in U$, $(\mu_{A\cap B}(u)=\min\{\mu_A(u),\mu_B(u)\})$, $\forall u\in U$, $(\mu_{A\cup B}(u)=\max\{\mu_A(u),\mu_B(u)\})$, and $\forall u\in U$, $(\mu_{\Theta A}(u)=1-\mu_A(u))$. Also, $A\subseteq B$ if and only if $\forall u\in U$, $(\mu_A(u)\leqslant\mu_B(u))$, $A=B$, if and only if $\forall u\in U$, $(\mu_A(u)=\mu_B(u))$. A uniquely determines the conjugate, ΘA. A and ΘA do not satisfy $A\cup\Theta A=$ Universe or $A\cap\Theta A=\varnothing$.*

Proof. By Theorem 5. □

Theorem 7. *Z_0-fuzzy set system in its simple form is the same as Zadeh's fuzzy set theory, i.e., Z_0-fuzzy* *set system is a new definition of Zadeh's fuzzy set theory.*

Proof. A fuzzy set, A, in both theories is $A = \{(u, \mu_A(u)) | \mu_A(u) \in [0,1], u \in U\}$. One is a theorem the other is a definition, and both satisfy all operators of Lemmas 2 and 4: $\forall u \in U$, $(\mu_{A\cap B}(u) = \min\{\mu_A(u), \mu_B(u)\})$, $\forall u \in U$, $(\mu_{A\cup B}(u) = \max\{\mu_A(u), \mu_B(u)\})$, and $\forall u \in U$, $(\mu_{\Theta A}(u) = 1 - \mu_A(u))$. Also, $A \subseteq B$ if and only if $\forall u \in U$, $(\mu_A(u) \leqslant \mu_B(u))$. A uniquely determines the conjugate, ΘA. A and ΘA do not satisfy $A \cup \Theta A =$ Universe and $A \cap \Theta A = \varnothing$. (Comment: in Zadeh's fuzzy set theory, set conjugate wrote Θ as $\neg$, is wrongly called as set complement.) Hence, both systems must satisfy the same formulas. Therefore, Z_0-fuzzy set system is a new definition of Zadeh's fuzzy set theory. □

Corollary 2. *Theorem* 5 *holds in Zadeh's fuzzy set theory. And* $D = \Theta A$ *if and only if* $\forall u \in U$, $(\mu_D(u) = \mu_{\Theta A}(u) = 1 - \mu_A(u))$, *where* ΘA *is the conjugate of* A.

4 Conclusion and Comparison

4.1 Comparison

Table 7 is a comparison of C-fuzzy set theory and Zadeh's set theory, where $\zeta_{B/A}(u) = \mu_{A\cap B}(u)/\mu_A(u)$ is the correlation coefficient of A's grade of membership on B's, $\zeta_{B/A}(u) + \zeta_{\neg B/A}(u) = 1$. $A = \{(u, Y_A(u), \mu_A(u)) | Y_A(u) \subseteq [0,1], \mu_A(u) = |Y_A(u)| \in [0,1], u \in U\}$ and $B = \{(u, Y_B(u), \mu_B(u)) | Y_B(u) \subseteq [0,1], \mu_B(u) = |Y_B(u)| \in [0,1], u \in U\}$ are two arbitrary C-fuzzy sets[6].

4.2 Conclusion

1) *Zadeh's Fuzzy Set's Shortcomings*: due to Zadeh's fuzzy set theory definitions, it is limited that all fuzzy sets must be "non-uniformly inclusive" (Theorem 5 and Corollary 2). Hence, it cannot correctly reflect different kinds of fuzzy phenomenon in the natural world, for example, in cases of "non-intersecting" or "intersecting but are not non-uniformly inclusive".

2) *Zadeh's Fuzzy Set's Mistakes*: it is incorrect to define the set complement as "$1 - \mu_A(u)$", because it can be proved that set complement may not exist in Zadeh's fuzzy set (Theorem 6). And it also leads to logical confusion, and is seriously mistaken to believe that logics of fuzzy sets necessarily go against classical and normal thinking, logic, and concepts.

3) *C-Fuzzy Set Theory*: it can be free of Zadeh's fuzzy set's shortcoming and mistake. All classical set relations and operators (including set complement $\neg$) exist and all formulas still hold, including $A \cup \neg A =$ Universe and $A \cap \neg A = \varnothing$ (Theorem 1). It is in accord with classical and normal thinking, logic, and concepts.

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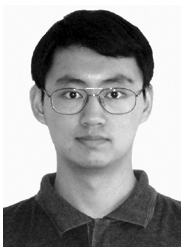

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