A Combinational Perspective in Stimulating Cooperation in Mobile Ad Hoc Networks

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Abstract   In wireless ad hoc networks cooperation among nodes cannot always be assumed since nodes with limited resources and different owners are capable of making independent decisions. Cooperation problems in topology control and packet forwarding tasks have been mostly studied separately but these two tasks are not independent. Considering a joint cooperation problem by taking into account dependencies between tasks will result in more reliable and efficient networks. In this paper topology control definition is extended to cover cooperation problem in both packet forwarding and topology control in a single problem. In this definition nodes have to adjust their transmission power and decide on their relay role. This paper models the interactions of nodes as a potential game with two-dimensional utility function. The presented model, named TCFORCE (Topology Control packet FORwarding Cooperation Enforcement), preserves the network connectivity and reduces the energy consumption by providing cooperative paths between all pairs of nodes in the network.

Keywords   game theory, non-cooperative, packet forwarding, topology control, wireless ad hoc networks

1   Introduction

1.1   Preliminaries

Wireless devices face incessant evolutions that empower them with more and more capabilities every day. These devices are nodes of the infrastructureless wireless ad hoc networks. The nodes are capable of making their own decisions, regarding the purpose of their different owners. Lack of centralized management in wireless ad hoc networks gives the nodes the opportunity to selfishly follow their own goals in the network. Selfish behavior of nodes is mainly due to the limited resources (energy and bandwidth). Therefore they may have no incentive to cooperate and consume their resources in the tasks on behalf of others. On the other hand cooperation among nodes is vital for network correct functionality. Selfish behavior of nodes may affect different layers of the network protocol stack and cause dramatically reduction in the network performance[1]. Cooperation stimulation in wireless ad hoc networks has been the subject of intensive research effort over the past decade.

Two main tasks in wireless ad hoc networks are topology control and packet forwarding. Packet forwarding is the main source of energy consumption in networks and selfish behavior of nodes in this task is inevitable. Topology control task assigns per node transmission power levels in order to achieve certain network wide goals such as connectivity and energy efficiency[2-3]. Selfish behavior of nodes in topology control may result in inefficient topologies for the network. Cooperation enforcement in topology control and packet forwarding has been mostly studied separately. Here we study the cooperation problem in these two related tasks in a single problem in order to capture the effect of their relationship on cooperation.

1.2   Motivation

Topology control and packet forwarding are not independent tasks. In wireless networks, in order to enable multi-hop routing, the underlying topology must not only be connected, but also it need to contain paths where intermediate nodes are willing to cooperate in forwarding packets. In other words, every pair of nodes in the network needs a path of cooperative nodes, in order to communicate. In performing topology control, nodes face temptations to conserve their total energy by reducing their power level and selecting closest

neighbors for relaying transmissions. However the concern is whether the selected neighbor is a relay node or not. It is quite unrealistic to assume (as many papers assumed, e.g., [4-5]) that network nodes act selfishly in forwarding task, while they are cooperative in topology control task. Even when we assume a cooperation enforcement mechanism in each task, we may not gain desirable efficiency due to neglecting the relationship between the tasks.

In the example provided in Fig.1 we show the effect of selfishness in packet forwarding on the topology control task. Suppose that node c is a selfish node in forwarding task and node b is a cooperative node.
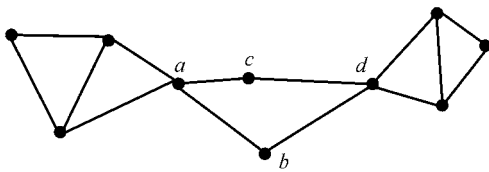

Fig.1. Neighbor selection example based on two metrics — power level and relay role.

In the topology control task if node a does not consider the selfishness of node c, it may choose its power level in a way that node c becomes its next hop in connecting the left and right parts of the network. Since we assumed node c is a selfish node, by this selection the resulted topology will consume more energy, cost or even it may get partitioned into two parts based on the level of selfishness. However if node a increases its power level to reach node b, it can alleviate the effect of node c in forwarding task and results in a more efficient and reliable topology. In this paper we provide a model that covers both of these tasks in a single topology control framework. Although this approach makes some complexities in the model but by considering these two related problems together we will be closer to an overall optimal solution.

We propose TCFORCE (Topology Control packet FORwarding Cooperation Enforcement) model that results in a more reliable and energy-efficient topology by considering cooperation of nodes in packet forwarding while establishing network topology. The problem is modeled as a potential game and it has been shown that this model will converge to connected, reliable, and energy-efficient network topologies. Our simulation results will demonstrate our claims. To the best of our knowledge this is the first paper that proposes a fully combinational perspective of topology control and packet forwarding in stimulating cooperation in mobile ad hoc networks (MANETs). The rest of the paper is structured as follows. We begin by providing a brief overview of related work in Section 2. An introduction

to game theory as applicable to our work is presented in Section 3. We discuss system model and assumptions in Section 4. This section also includes the network model, energy model and TCFORCE game model. We analyze our game model in Section 5. Section 6 provides TCFORCE game theoretic algorithm for stimulating cooperation in topology control and packet forwarding. Simulation results are presented in Section 7 and Section 8 outlines future work and conclusions.

2 Related Work

Cooperation enforcement in routing and packet forwarding, and topology control are two main fields in studying the non-cooperative behaviors of nodes in wireless ad hoc networks. Many papers have been focused on improving cooperation in packet forwarding and routing tasks[4-9]. Totally, there are two principal classes of cooperation enforcement mechanisms in routing and packet forwarding tasks: reputation-based[7-8] and price-based[4-6] mechanisms. In reputation-based approaches nodes assign a reputation to other nodes based on their observed behavior. Nodes with higher reputation will be better served in the network. These approaches mainly suffer from large overhead and need of trust relationships between nodes. In price-based mechanisms, nodes gain some kind of virtual money if they cooperate to provide services to others and they have to pay to other nodes that provide services to them. These approaches need some secure mechanisms to make them applicable in real systems. In topology control task, nodes selfishly attempt to minimize their energy consumption by reducing their transmission power while remaining connected to the network. This behavior may result in inefficient topologies. Considering the impact of selfish nodes in topology control is a quite recent research thread[10-14]. The work presented in [10] provides an example when existing topology control approaches are not resilient to possible selfish node behavior and claim that many topology control protocols proposed till now have weak point for selfish nodes to use. In this work an incentive compatible protocol is proposed for topology control which induces nodes to cooperate with virtual money, similar to price-based mechanisms in forwarding task. In [14] a truthful topology control mechanism (TRUECON) is proposed to induce the selfish network nodes to collaborate. The mechanism is a cone-based topology control algorithm and uses a VCG mechanism to make the truth-telling dominant strategy. We cannot put [14] exactly in the cooperation stimulating mechanisms in topology control games, because this model uses topology information to specify the cost of each of neighbors in forwarding packets according to the power needed to

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reach that neighbor. Other work that cannot be placed exactly in one of these categories is [15] in which a strategy proof mechanism for constructing a topology of relay node is proposed and gives enough incentive to the nodes to make them cooperate in forwarding packets. This approach is a price-based approach in constructing a topology for forwarding packets. This paper can be categorized in cooperation stimulating mechanisms in packet forwarding task with price-based mechanism too. The presented work in [13] models the topology control game as a potential game which guarantees the existence of equilibrium and provides the best (namely MIA) and better response (namely DIA) algorithms for nodes to choose their transmission power level. [11] mainly studies Nash Equilibrium (NE) properties of different topology control games. In [12] authors have added the impact of packet forwarding cooperation level (probability of forwarding packets by nodes) to the utility function in the topology control game beside other factors presented in [13] (e.g., energy consumption) in deciding the power level of the nodes. They assumed that selfish behavior of nodes in packet forwarding can be express by the percentage of packets a node forward for others. They assumed that all the packet forwarding levels are known exogenously. The work in [12] is the only approach which tries to consider the impact of selfish packet forwarding on the topology control but one should note that the level of cooperation of the nodes are assumed to be known and cooperation in forwarding is assumed to be achievable with some mechanism. Also we will discuss later that the energy model presented in [12] is not realistic. Works in [12-13] are the closest works to this paper and we will try to discuss them more and compare our results with them.

3 Game Theory and Potential Games

In this section we present a brief overview (summarized by [12]) of game theory[16-17] that is applicable to our work. Our focus will be on potential games[18].

Game is a formal model of an interactive decision making situation. Strategic non-cooperative game $\Gamma = \langle N, A, \boldsymbol{u} \rangle$ has three components.

• Player set, N which contains the n players of the game where n is the number of players.

• Action set $A : \boldsymbol{a} \in A = \times_{i=1}^{n} A_i$, where A_i is the space of all action vectors, where each component, a_i, of the vectors $\boldsymbol{a}$ belongs to the set A_i which is the set of actions of player i. Often we denote an action profile $\boldsymbol{a} = (a_i, \boldsymbol{a}_{-i})$, where a_i is the player i's action and $\boldsymbol{a}_{-i}$ is the action vector of other $n - 1$ players.

• For player $i \in N$, utility function $u_i : A \to \mathbb{R}$ models the preferences of player i over the action profiles. Vector of such utility functions is denoted by

$$\boldsymbol{u} = (u_1, \ldots, u_n) : A \to \mathbb{R}^n.$$

The most common solution for games is the Nash Equilibrium (NE). NE is a stable point where no player has any incentive to unilaterally deviate from his action. NE in some sense is a consistent predictor of possible outcomes of a game.

Definition 1. *An action profile $\boldsymbol{a}^* = (a_i^*, \boldsymbol{a}_{-i}^*)$ is a Nash Equilibrium if, $\forall a_i \in A_i$,*

$$u_i(\boldsymbol{a}^*) \geqslant u_i(a_i, \boldsymbol{a}_{-i}^*). \tag{1}$$

A game may possess a large number of Nash Equilibrium or none at all. Some classes of games are known to posses at least one NE.

Definition 2. *A strategic game $\Gamma = \langle N, A, \boldsymbol{u} \rangle$ is an ordinal potential game (OPG) if there exists a function $V : A \to \mathbb{R}$ such that $\forall \boldsymbol{a}_{-i} \in A_{-i}$ and $\forall a_i, b_i \in A_i$*

$$V(a_i, \boldsymbol{a}_{-i}) - V(a_i, \boldsymbol{a}_{-i}) > 0 \Leftrightarrow$$
$$u_i(a_i, \boldsymbol{a}_{-i}) - u_i(a_i, \boldsymbol{a}_{-i}) > 0, \tag{2}$$

where V is called the ordinal potential function (OPF) of Γ. In essence, in an OPG, utility function of players can be replaced with the OPF, because OPF exhibits the same "directional" behavior, when individual players unilaterally deviate. Thus pure strategy equilibrium set of the game coincides with the pure strategy equilibrium set of the game in which every player's utility is given by OPF. This model simplifies the analyzing of the game.

Potential games with compact action spaces are known to possess at least one NE in pure strategies[18]. The following lemma[18] establishes how Nash Equilibrium of the game can be identified.

Lemma 1. *Let Γ be an OPG and V its corresponding OPF. If $\boldsymbol{a} \in A$ maximizes V, then it is an NE.*

If we identify potential functions for a game, we can immediately identify some NE of the game by solving for potential maximizers.

4 System Model and Assumptions

4.1 Network Model

We model the network as a set of mobile and wireless nodes with omnidirectional antennas, which are embedded in a 2D plane region. Nodes have the ability to adjust their transmission power to a power level lower than the maximum default one. For example we assume that nodes are equipped with the network cards, like Cisco Aironet 350 series cards (IEEE 802.11b compliant) which allow the transmit power level to be set to one of 1, 5, 20, 30, 50, 100 mW level[19].

We also assume some level of uncertainty in wireless communications due to their nature. This necessitates the need of link level acknowledgement for packet

received. Hence we assume bidirectional links in our network model.

The network topology is modeled as a graph $G = (N, E, \boldsymbol{\Omega})$ where N is the set of nodes and there are n nodes in the set, $E \subseteq N^2 = N \times N$ is the set of directed arcs representing unidirectional connections. The weight matrix $\boldsymbol{\Omega} = [w_{ij}]$ assigns weights to links. There are many parameters that may affect this weight (e.g., gain loss, thermal noise, SNR), but here for simplicity we assume that weight of a link is proportional to the transmission power level needed to establish that link. The power level needed to establish a link is corresponding to the Euclidean distance between the nodes. This means that there will be a link between nodes i and j if $\boldsymbol{P}(j) = p_j \geqslant w_{ij}$ and $\boldsymbol{P}(i) = p_i \geqslant w_{ij}$ where $\boldsymbol{P}$ is the vector of power levels of the nodes.

We have defined two subsets of nodes in our model. The set of relay nodes ($\mathcal{C}$) which will relay packet for other nodes in the network, and the set of non-relay or selfish nodes ($\mathcal{S}$) which will not forward any data packet for others, ($\mathcal{C} \cup \mathcal{S} = N$).

As mentioned before $\boldsymbol{P} = (p_1, \ldots, p_n)$ is the power level vector of all nodes in the network. Furthermore $\boldsymbol{F} = (f_1, \ldots, f_n)$ is the relay role vector where

$$f_i = \begin{cases} 1, & \text{if } i \text{ is a relay node,} \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

According to these assumptions we define two perspectives in the network topology formation. The first perspective is the traditional definition of topology control which tries to assign appropriate power level to the nodes while preserving some network wide goals such as connectivity and energy-efficiency. The second one is a new aspect of topology control in the presence of the selfish nodes which tries to assign relay roles to nodes while preserving the same network wide goals. This approach is constructing a relay backbone for the network with selfish behavior in forwarding task. Therefore in this network model, topology of the network is represented by $G(\boldsymbol{P}, \boldsymbol{F})$. We define $G_{\max}$ to be the network in which all nodes are relay nodes and they transmit with their maximum power level, $P_i^{\max}$ where $P_i^{\max}$ is the maximum power level node i can choose.

We define two sets of reachability for each node in the network. First direct reachability set or neighbor set of node i, $N_i(p_i)$, is the set of nodes that have bidirectional link with node i. We define path ρ_{ij} between two nodes i and j, such that every node on the path is in the neighbor set of its previous and next hop nodes on the path.

Second, indirect reachability set of node i, $D_i(\boldsymbol{P}, \boldsymbol{F})$, is the set of all nodes that are reachable over multi-hop cooperative paths from node i.

$$D_i(\boldsymbol{P}, \boldsymbol{F}) = \{j | j \notin N_i(p_i), \text{ and } \exists \rho_{ij} \in G(\boldsymbol{P})$$
$$\text{where} \quad \forall k \in \rho_{ij}, \ f_k = 1, \text{ and } i, j \in N\}. \quad (4)$$

Now we present the *Connectivity* definition in the network, on top of other definitions. Network $G(\boldsymbol{P}, \boldsymbol{F})$ is connected if and only if

$$\forall i, j \in N : i \in D_j(\boldsymbol{P}, \boldsymbol{F}) \text{ or } i \in N_j(p_j). \quad (5)$$

These definitions are for the purpose of modeling our problem, and may differ slightly from topology control and graph theory terminologies.

4.2 Energy Model

Energy consumptions of nodes depend on the power levels they use to connect to the network. This power level determines the transmission range and neighbor set of the node. We also assume that nodes utilize per-packet power control approach which has been shown to be an effective power control strategy in networks[20]. Based on our network model, the energy that a node will consume for forwarding data packets in a network has two parts. The energy consumed to send its own data packets and the energy consumed to forward other data packets, if the node is a relay node. If λ_i shows the rate of traffic generation in node i and λ the average traffic rate passes through a node in the network, we can approximate the energy consumption for data transmission of a node as

$$\mathcal{E}_i^d(p_i, f_i) = \lambda_i p_i + \lambda f_i p_i. \quad (6)$$

Nodes send control messages with their maximum power. We assume that this energy is not negligible because of maximum power used and large overhead of control messages. Thus, if we assume the control message rate c, the energy consumption of control message is approximately

$$\mathcal{E}_i^c(p_i) = c p_i, \quad (7)$$

and the total energy consumption of a node will be

$$\mathcal{E}_i^t(p_i, f_i) = (c + \lambda) p_i + \lambda f_i, p_i. \quad (8)$$

This energy model is much more realistic compared with the model presented in [12], and the energy of receiving packets can easily be added to this model without any change in the problem. The energy model presented in [12] has neglected the reception cost although it is not negligible in the real networks based on [21]. If we add the reception cost to the energy model of [12], we will have a contradiction in the concept of

the problem. The model in [12] considers retransmission. This way selfish node puts force on its previous node to retransmit dropped data packets because of its selfishness but this will impact the reception cost of the selfish node too. We should notice that nodes are selfish to save their resources like energy and bandwidth but the model in [12] has punishment in its nature for selfishness and will increase energy consumption of selfish node and also total network energy consumption. Beside the energy aspects, retransmissions also affect transmissions delay and increase the chance of collision for node's transmissions. In other words, the selfishness in this energy model is in contrast with the goals of selfish node.

We define some energy efficiency metrics to evaluate energy efficiency of the generated topologies. First we define the energy consumed in a path ρ_{ij} as

$$\mathcal{E}_{\rho_{ij}} = \sum_{k \in \rho_{ij}, k \neq j} \bar{\mathcal{E}}_k. \tag{9}$$

Here, $\bar{\mathcal{E}}_k = w_{kl}$ is the energy consumed by node k in the path, and $\rho_{ij}^{\min}$ is called a minimum energy path if it consumes the least amount of energy to transmit packets from node i to j. We define average minimum path cost between every pairs of nodes in the network as

$$\mathcal{E}_{\rho^{\min}} = \Big(\sum_{i \in N} \sum_{i \neq j} \mathcal{E}_{\rho_{ij}^{\min}} \Big) \Big/ (n \times (n-1)/2). \tag{10}$$

An energy efficient protocol is said to have minimum energy property if it preserve the minimum energy paths between every source-destination pair[22]. Based on the definition presented in [13], a connected network is said to be globally energy efficient if $\mathcal{E}_{\boldsymbol{P}} = \sum_{i=1}^{N} p_i$ is minimized. Also a connected network is said to be locally energy efficient if no node can reduce its transmission power level without disconnecting the network.

4.3 TCFORCE Game Model

In this subsection we design the game model in a way that the outcome of the game coincides with the desirable outcome, which is cooperation in packet forwarding and topology control. In this model, nodes play and compete to choose their transmission power level and relay role in the network. Nodes selfishly choose their transmission power level as low as possible. They also try to assign the relay role to other nodes of the network. The topology control is modeled as a noncooperative game. We define our game, $\Gamma = \langle N, A, \boldsymbol{u} \rangle$ as follows.

• Player set, N with is the set of n nodes in the network;

• Action set $A : \boldsymbol{a} \in A = \times_{i=1}^{n} A_i$, where A_i is a Cartesian set of two sets, i.e., $A_i = p_i \times f_i$, where $p_i \in [0, 1]$ is the power levels and $f_i \in \{0, 1\}$ is the relay roles. Therefore nodes have to choose their power level and relay role as their action in each step of game and use that action profile till next topology control game.

• For each player $i \in N$, a two-dimensional utility function $u_i(\boldsymbol{P}, \boldsymbol{F}) : A \to \mathbb{R}$ is provided which models nodes preferences over the action profiles. This utility function will be described in more detail in the following section.

In the next section we will prove that the presented game model is an ordinal potential game. The outcome of the game in each iteration is a power vector $\boldsymbol{P} = (p_1, \ldots, p_n)$ and a relay role vector $\boldsymbol{F} = (f_1, \ldots, f_n)$ (which together will specify the topology of the network). An appropriate power level and relay role selection of a node depend on other nodes' power levels and relay roles. Each node perceives a trade-off between the benefit it derives from topology and the cost it incurs. The cost that a node has to pay in a topology is the energy it will consume in that topology. We assume that all the nodes in the network want to be connected to all the other nodes, because they do not know the destination of their future communications. Thus one benefit from the topology is the connectivity. We define another benefit a node can derive from a topology named *priority*. As mentioned in [18], an improvement path of actions in a potential game will converge to a Nash Equilibrium. In any path of actions in the game, just one node can change its action. The order nodes take to choose their action in a potential game in an improvement path of actions, may result in different Nash Equilibrium points. Nodes that have the opportunity to choose their action earlier than others have better options for their choice. In this model, nodes with high priority can choose their action earlier and have the chance to choose lower power levels and a higher chance of not being a relay node. Hence, higher priority in the next topology control game is a benefit a node can derive from the topology. Later we will discuss priority in more detail. We cast these interdependencies in a utility function for each node i as

$$u_i(\boldsymbol{P}, \boldsymbol{F}) = \alpha \times Z_i(\boldsymbol{P}, \boldsymbol{F}) + Q_i^{TC+1}(\boldsymbol{P}, \boldsymbol{F}) - \mathcal{E}_i^t(p_i, f_i). \tag{11}$$

In (11), the term α is a fractional scalar and will specify the importance of first term in the utility function (we will discuss this term later). $Z_i(\boldsymbol{P}, \boldsymbol{F})$ is the number of nodes that can be reached by node i and is defined to be

$$Z_i(\boldsymbol{P}, \boldsymbol{F}) = |D_i(\boldsymbol{P}, \boldsymbol{F})| + |N_i(p_i)|. \tag{12}$$

In (12), the the cardinality of a set A is shown by $|A|$. In (11) $\mathcal{E}_i^t(p_i, f_i)$ has been introduced in the energy model and is the energy that a node will consume in the network based on the topology. The second term is the priority of the node in choosing its action in the next topology control game and is defined as

$$Q_i^{TC+1}(\boldsymbol{P}, \boldsymbol{F}) =$$
$$\begin{cases} M + \alpha_1 p_i + \beta_1 f_i N_i(p_i), & \text{if } x_i(\boldsymbol{P}, \boldsymbol{F}) \geqslant x_i^{-1}(\boldsymbol{P}, \boldsymbol{F}), \\ 0, & \text{otherwise,} \end{cases} \quad (13)$$

where $x_i(\boldsymbol{P}, \boldsymbol{F}) = \left(\sum_{j \in N_i(p_i)} Z_i(\boldsymbol{P}, \boldsymbol{F})\right)/|N_i(p_i)|$ and $x_i(\boldsymbol{P}, \boldsymbol{F})$ is the average reachability of neighbors of node i after its selection of relay role and power level. Also $x_i^{-1}(\boldsymbol{P}, \boldsymbol{F})$ is the average reachability of neighbors before node i's selection. This shows that how the decision of node i affects other nodes in the neighborhood. If node i's decision causes any reduction in the reachability of the others then its priority will set to zero, otherwise it will gain a priority corresponding to the power level and relay role it had chosen. Term M, α_1 and β_1 are fractional scalars which with the term α mentioned previously, balance the importance of different terms in the utility function (11). Terms M and α signify the importance each node places on being connected and the priority it gains for the next topology control game respectively. Table 1 represents the appropriate range for these terms. These ranges are obtained based on the proof presented in the next section which shows this game model is an ordinal potential game. We will discuss the role of these terms in the next section. Fig.2 shows the impact of priorities on the power level selections. This result shows that on average higher priority (earlier turns) will results in lower power levels for the nodes.

Table 1. Appropriate Range of Terms in Utility Function (11)

Terms	Appropriate Range
α	$\alpha > M \times p_i^{\max}$
M	$M \geqslant \mathcal{E}_i^{\max}$
α_1	$0 < \alpha_1 < (\lambda_i^{\max} + c)$
β_1	$(\lambda_i^{\max} + c) < \beta_1 \leqslant \lambda \times p_i^{\max}$

Therefore having the opportunity to choose the power level earlier is a good incentive for the nodes to cooperate. The wave form plot (large dot plot) is for a network with 50 nodes with randomly assigned priorities. This plot shows the power levels selected by nodes which are sorted based on their turn in the game. The oscillations are due to the impact of selections of other nodes and also the position of the node in the network. The average over 50 topologies consisting of 50 nodes and 20 different turn permutations of the nodes (little

dot plot) shows very well that the power level is an increasing function of the turns.

Fig.3 represents the chance of nodes in not being a relay node in different priorities (turns). As Fig.3 shows, nodes with higher priority (lower turns) have better chance in not being a relay node. In lower priority nodes have to choose relay role with higher probabilities. The simulation settings are the same as the case of Fig.2.
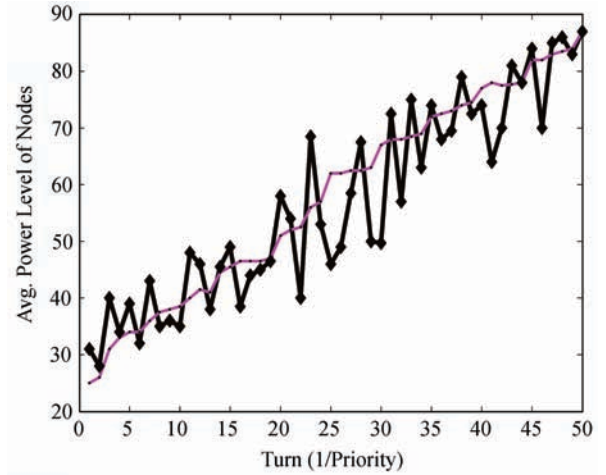

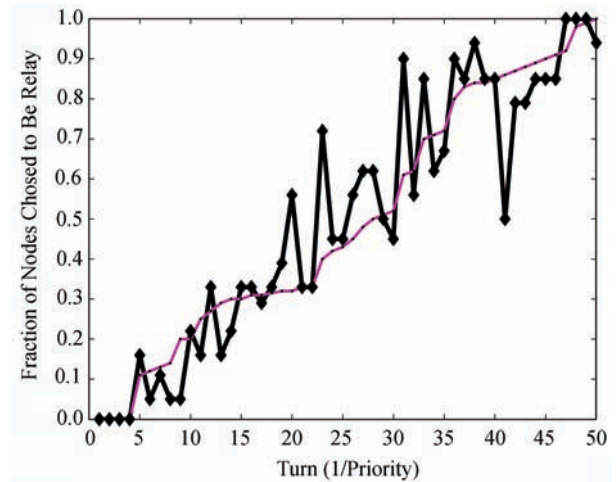Fig.2. Node priorities impact on the power level selection.

Fig.3. Node priority impact on the relay role selections.

Therefore priority is added to the utility function of nodes as the benefit they can derive from the topology they contribute in constructing it. We also choose $\alpha_1 < \beta_1$ (relay role have greater impact on the priority) to give the higher incentives to nodes that are cooperative in packet forwarding. Other bounds for these terms have chosen such that energy consumption of the nodes has greater impact on the utility function. We present topology control algorithm based on presented model.

5 Game Theoretic Analysis

In this section we first show that the game $\Gamma = \langle N, A, \boldsymbol{u} \rangle$ with the utility function given by (11) is an ordinal potential game and then we will discuss some features of this game.

Theorem 1. *Game $\Gamma = \langle N, A, \boldsymbol{u} \rangle$, where the individual utilities are given by (11), is an OPG. The OPF of this model is given by*

$$V(\boldsymbol{P}, \boldsymbol{F}) = \alpha \times \sum_{i \in N} Z_i(\boldsymbol{P}, \boldsymbol{F}) + \sum_{i \in N} Q_i^{TC+1}(\boldsymbol{P}, \boldsymbol{F}) - \sum_{i \in N} \mathcal{E}_i^t(p_i, f_i). \tag{14}$$

We prove this claim by applying the asserted OPG in (14). We have

$$\begin{aligned} \Delta u_i &= u_i(a_i, \boldsymbol{a}_{-i}) - u_i(\acute{a}_i, \boldsymbol{a}_{-i}) \\ &= \alpha \big[Z_i(a_i, \boldsymbol{a}_{-i}) - Z_i(\acute{a}_i, \boldsymbol{a}_{-i}) \big] + \\ & \quad \big[Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) - Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}) \big] - \\ & \quad \big[\mathcal{E}_i^t(a_i, \boldsymbol{a}_{-i}) - \mathcal{E}_i^t(\acute{a}_i, \boldsymbol{a}_{-i}) \big]. \end{aligned} \tag{15}$$

We denote again that $\boldsymbol{a} = (a_i, \boldsymbol{a}_{-i})$ is the profile strategy of the game, and $a_i = (p_i, f_i)$, is the profile strategy of node i. A node may deviate from its action in three ways compared with $a_i = (p_i, f_i)$, represented as follows.

$$\acute{a}_i = \begin{cases} (q_i, f_i), \\ (p_i, g_i), \\ (q_i, g_i). \end{cases} \tag{16}$$

Similarly for ΔV, we have

$$\begin{aligned} \Delta V_i &= V_i(a_i, \boldsymbol{a}_{-i}) - V_i(\acute{a}_i, \boldsymbol{a}_{-i}) \\ &= \alpha \big[Z_i(a_i, \boldsymbol{a}_{-i}) - Z_i(\acute{a}_i, \boldsymbol{a}_{-i}) \big] + \\ & \quad \big[Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) - Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}) \big] - \\ & \quad \big[\mathcal{E}_i^t(a_i, \boldsymbol{a}_{-i}) - \mathcal{E}_i^t(\acute{a}_i, \boldsymbol{a}_{-i}) \big] + \\ & \quad \alpha \sum_{j \in N, j \neq i} \big(Z_i(a_i, \boldsymbol{a}_{-i}) - Z_i(\acute{a}_i, \boldsymbol{a}_{-i}) \big) + \\ & \quad \sum_{j \in N, j \neq i} \big(Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) - Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}) \big) - \\ & \quad \sum_{j \in N, j \neq i} \big(\mathcal{E}_i^t(a_i, \boldsymbol{a}_{-i}) - \mathcal{E}_i^t(\acute{a}_i, \boldsymbol{a}_{-i}) \big). \end{aligned} \tag{17}$$

The fifth and sixth terms in (17) are equal to zero, because node i's choices will not affect other nodes' priority and energy consumption thus

$$\Delta V_i = \Delta u_i + \alpha \sum_{j \in N, j \neq i} \big(Z_i(a_i, \boldsymbol{a}_{-i}) - Z_i(\acute{a}_i, \boldsymbol{a}_{-i}) \big). \tag{18}$$

According to Table 1 and equations mention till here we find the sign of Δu_i in different cases of (16). Here is

where the range of values presented in Table 1 come to play to specify the sign of Δu_i and in different cases of (16). In these ranges of values we see that the combination of the linear terms in utility function and potential function with Table 1 values as their coefficient will result in $sgn(\Delta \boldsymbol{V}) = sgn(\Delta \boldsymbol{u})$ and therefore an ordinal potential game model.

In the first case of (16) a node just changes its power level $\acute{a}_i = (q_i, f_i)$. Thus in the first case and by considering Table 1 we have

$$\Delta u_i = \begin{cases} > 0, & \text{if } p_i > q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) > Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ < 0, & \text{if } p_i < q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) < Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ < 0, & \text{if } p_i > q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ > 0, & \text{if } p_i < q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \end{cases} \tag{19}$$

and in each corresponding case of (19)

$$\begin{cases} Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \geqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \leqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \geqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \leqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}). \end{cases} \tag{20}$$

As the sign of the second term in (18) is the same as first two cases of (19) and its value is zero for the third and forth cases of (19), we conclude $sgn(\Delta \boldsymbol{V}) = sgn(\Delta \boldsymbol{u})$.

For the second case of (16), node i changes its relay role, $\acute{a}_i = (p_i, g_i)$. In this case there will be two sub cases.

(a) $g_i = 0, f_i = 1$. In this case we have $Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i})$. There are two cases. If node's choice of not being relay node reduces other nodes' reachabilities and therefore $Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) = 0$ thus $\Delta u_i > 0$. On the other hand the sign of second term in (18) in this case will be positive. Thus we have $sgn(\Delta \boldsymbol{V}) = sgn(\Delta \boldsymbol{u})$. However if node's choice of not being a relay node does not affect other nodes' reachability, the second term of (18) will be zero. Thus $sgn(\Delta \boldsymbol{V}) = sgn(\Delta \boldsymbol{u})$.

(b) $g_i = 1, f_i = 0$. In this case we have $Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i})$. As before there are two cases. If node's choice of being a relay node increases other nodes' reachabilities thus $Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) < Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i})$, and therefore $\Delta u_i < 0$. On the other hand the sign of second term in (18) is negative and $sgn(\Delta \boldsymbol{V}) = sgn(\Delta \boldsymbol{u})$. However if node's choice of being relay node does not affect other nodes' reachability, the second term of (18) will be zero and $sgn(\Delta \boldsymbol{V}) = sgn(\Delta \boldsymbol{u})$.

In the third case of (16), node i changes both its power level and relay role. We consider this case in two sub cases as previous.

(a) $g_i = 0, f_i = 1$. If the node's choice of not to be a relay node does not affect other ones reachability we

have

$$\Delta u_i = \begin{cases} > 0, & \text{if } p_i > q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) > Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ < 0, & \text{if } p_i < q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) < Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ < 0, & \text{if } p_i > q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ > 0, & \text{if } p_i < q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \end{cases} \tag{21}$$

and in each corresponding case of (21)

$$\begin{cases} Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \geqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \leqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \geqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \leqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}). \end{cases} \tag{22}$$

The second term of (18) has the same sign as the first two cases of (21). For the last two cases of (21), the second term of (18) is zero or positive. Thus we have $sgn(\Delta \boldsymbol{V}) = sgn(\Delta \boldsymbol{u})$. However if node's choice of being a relay node changes other nodes' reachability, we have

$$\Delta u_i = \begin{cases} > 0, & \text{if } p_i > q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) > Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ > 0, & \text{if } p_i < q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) < Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ > 0, & \text{if } p_i > q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ > 0, & \text{if } p_i < q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \end{cases} \tag{23}$$

and in each corresponding case of (23)

$$\begin{cases} Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \geqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \geqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \geqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \geqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}). \end{cases} \tag{24}$$

As we assume reduction in the other nodes' reachability, the sign of the second term of (18) will be positive for all the cases, and thus we have $sgn(\Delta \boldsymbol{V}) = sgn(\Delta \boldsymbol{u})$.

(b) $g_i = 1, f_i = 0$. If nodes' choice of being relay node does not affect other ones reachability we have

$$\Delta u_i = \begin{cases} > 0, & \text{if } p_i > q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) > Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ < 0, & \text{if } p_i < q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) < Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ < 0, & \text{if } p_i > q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ > 0, & \text{if } p_i < q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \end{cases} \tag{25}$$

and in each corresponding case of (25)

$$\begin{cases} Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \geqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \leqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \geqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \leqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}). \end{cases} \tag{26}$$

The second term of (18) has the same sign as the first two cases of (26). For the last two cases of (26),

the second term of (18) is zero or negative. Therefore $sgn(\Delta \boldsymbol{V}) = sgn(\Delta \boldsymbol{u})$. However if node's choice of being relay node increases other nodes reachability

$$\Delta u_i = \begin{cases} > 0, & \text{if } p_i > q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) > Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ < 0, & \text{if } p_i < q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) < Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ < 0, & \text{if } p_i > q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \\ < 0, & \text{if } p_i < q_i \text{ and } Z_i(a_i, \boldsymbol{a}_{-i}) = Z_i(\acute{a}_i, \boldsymbol{a}_{-i}), \end{cases} \tag{27}$$

and in each corresponding case of (27)

$$\begin{cases} Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) < Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) < Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \leqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}), \\ Q_i^{TC+1}(a_i, \boldsymbol{a}_{-i}) \leqslant Q_i^{TC+1}(\acute{a}_i, \boldsymbol{a}_{-i}). \end{cases} \tag{28}$$

As we assume increasing in the other nodes' reachability, the sign of the second term of (18) will be negative for all the last three cases of (27), but for the first case of (27) although node becomes a relay node but choosing lower power level reduces other nodes' reachability and thus the sign of the second term in (18) is positive and thus $sgn(\Delta \boldsymbol{V}) = sgn(\Delta \boldsymbol{u})$. Therefore based on Definition 1 presented in Section 3, sign changes of the utility function and potential function are same therefore V is an OPF and Γ is an OPG. We can conclude that this game will have at least an NE, which is the OPF minimizer based on Lemma 1.

Theorem 2. *The potential maximizer of the game* $\Gamma = \langle N, A, \boldsymbol{u} \rangle$, *preserves network connectivity.*

Proof. We prove by contradiction. Suppose that $\acute{o} = (\boldsymbol{P}, \boldsymbol{F})$ is the OPF minimizer and o is a point that preserve network connectivity. As $V(\acute{o}) > V(o)$, we have

$$\alpha \times \sum_{i \in N} Z_i(\acute{o}) + \sum_{i \in N} Q_i^{TC+1}(\acute{o}) - \sum_{i \in N} \mathcal{E}_i^t(\acute{o}) >$$
$$\alpha \times \sum_{i \in N} Z_i(o) + \sum_{i \in N} Q_i^{TC+1}(o) - \sum_{i \in N} \mathcal{E}_i^t(o). \tag{29}$$

Based on our assumption $\acute{o}$ will not result in connected topology thus

$$Z_i(o) = n - 1, \quad Z_i(\acute{o}) = k_i < n - 1, \tag{30}$$

and we can rewrite (29) as

$$\sum_i (n - 1 - k_i) < \sum_i \left(Q_i^{TC+1}(\acute{o}) - Q_i^{TC+1}(o) \right) +$$
$$\sum_i \left(\mathcal{E}_i^t(o) - \mathcal{E}_i^t(\acute{o}) \right), \tag{31}$$

where $\sum_i (\mathcal{E}_i^t(o) - \mathcal{E}_i^t(\acute{o})) \leqslant n\mathcal{E}_i^{\max}$, and also $\sum_i (Q_i^{TC+1}(\acute{o}) - Q_i^{TC+1}(o)) < 0$, because some nodes have

reduced the reachability of the others. The right hand side of (31) is thus smaller than $n\mathcal{E}_i^{\max}$. According to Table 1, $\alpha \geqslant M \times p_i^{\max}$ so the left hand side of (31) is larger than $n\mathcal{E}_i^{\max}$ and this is a contradiction. $\square$

Theorem 3. *For the game $\Gamma = \langle N, A, \boldsymbol{u} \rangle$, the class of potential minimizers coincide exactly with class of topologies that are locally energy efficient.*

Proof. Suppose that NE is not locally energy efficient. Thus there exists a node i that can further reduce its power without disconnecting from the network or reduce others reachability. Based on Table 1, energy consumption has greater impact on the utility of a node than the priority it gains by higher power levels. Since nodes are rational and prefer higher utilities, that point is not an NE. $\square$

6   TCFORCE Algorithm

Based on the game model introduced in the previous section, we propose a TC algorithm called TCFORCE, for topology formation in the presence of selfish nodes. In TCFORCE algorithm nodes adapt their transmission power level and relay role, according to a greedy best response process. Based on the TCFORCE game model, we formalize the three phases of this algorithm similar to [13]. Totally time is broken into two periodic intervals, TCFORCE algorithm execution phase which specifies topology by specifying power levels and relay role of nodes, and data transferring phase which routing and data forwarding will be done over the specified topology $G(\boldsymbol{P}, \boldsymbol{F})$, based on power vector $\boldsymbol{P} = (p_1, \ldots, p_n)$ and relay role vector $\boldsymbol{F} = (f_1, \ldots, f_n)$. TCFORCE algorithm consists of three phases, initialization phase, adaptation phase, and update phase. We next present these phases in detail.

6.1   Initialization Phase

In this phase each node transmits a beacon with its maximum power ($P_i^{\max}$) to discover its neighbor set. By collecting acknowledgements from its neighbors, neighbor set will be found. At the end of this phase each node will have a neighbor table in which neighbors, the power required to reach them, and the relay roles of them (at the beginning all the nodes assumed to be relay) are represented. These neighbor sets will represent our initial topology ($G_{\max}$).

6.2   Adaptation Phase

In this phase nodes determine their transmission power level and their relay role in the topology based on the utility function provided in (11). As we mentioned previously, only one node adapts its settings at a time. The node that can adapt its settings is chosen

based on its priority. We assume that all the nodes have a priority-based randomized timer. This timer generates a random time based on the priority of the node. The higher the priorities, the smaller amount of time needed to wait. Nodes can update their strategies whenever the timer goes off. We use normalized priorities between [0, 1] based on (13). For normalizing nodes' priority for next execution of TCFORCE algorithm, we neglect term M in (13). For the first execution of the TCFORCE algorithm and for new arrival nodes to the network we use 0.5 as priority of nodes. We also assume that the priority based randomized timer is a tamper proof hardware and cannot be modified by nodes. These kinds of timers are common in MAC layer protocols for generating back off interval time for avoiding collision[23]. This way of choosing nodes is not the only way, but this is reasonably justified because in a practical setting the probability of any two nodes updating their strategies at the same time instant is zero[13].

Each iteration of the game can be viewed as a normal form game, wherein every node chooses to maximize utility in that iteration. This iterative process allows network to evolve dynamically. In TCFORCE algorithm whenever a node has the chance to revise its settings, it does its revises based on

$$(p_i, f_i) = \max_{(q_i, g_i) \in A_i} u_i((q_i, g_i), (\boldsymbol{P}_{-1}, \boldsymbol{F}_{-1})). \qquad (32)$$

6.3   Update Phase

Whenever a node in the adaptation phase chooses a power level and a relay role, this choice will redefine its neighbor set and in turn modifies the overall topology. Once a particular node adapts its power level and relay role in the current topology it broadcasts these choices information. By receiving these control messages other nodes update their respective neighbor sets. In turn, the nodes respond to the topology changes and other nodes' choice of strategies by choosing an power level and relay role. If none of the nodes update the strategies from the current strategy that means TCFORCE has converged to a steady state (NE).

As our game model is a potential game it assures the existence of NE and thus TCFORCE algorithm converges to an NE after some iterations.

7   Simulation Results

We present simulation results to demonstrate the validity of TCFORCE model. We implemented TCFORCE algorithm in C++ with GUI in MATLAB. In our experiments, network nodes are distributed uniformly into a $1000\,\text{m} \times 1000\,\text{m}$ area. For simplicity the

power required to support a link ij is $w_{ij} = p_{ij} = d_{ij}^2$ where d_{ij} is the Euclidean distance between node i and j. In Fig.4 we consider the initial state topology, $G_{\max}$, containing 50 nodes with each node transmitting at $P_i^{\max} = 100\,\text{mW}$ and the initial topology is connected. Here filled circles denote relay nodes in the network.
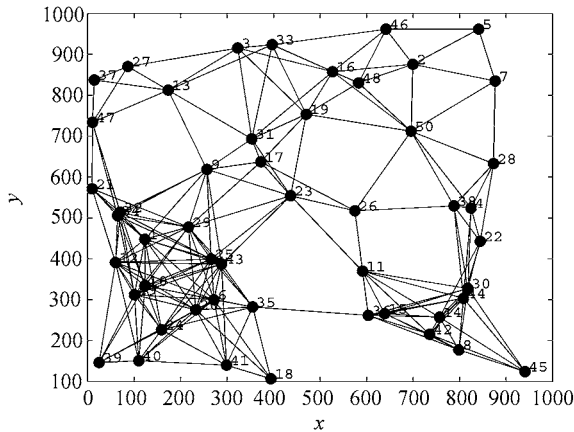

Fig.4. Initial network topology.

In Fig.5 an NE of TCFORCE algorithm for the initial topology presented in Fig.4 is shown. In this figure empty circles represent non relay nodes. It is obvious that TCFORCE algorithm has converged to a connected topology based on the definition of our network model and there is a path of relay nodes between every pair of nodes in the network. We assumed nodes reduce their transmission power to one of 1, 5, 20, 30, 50, 100 mW.
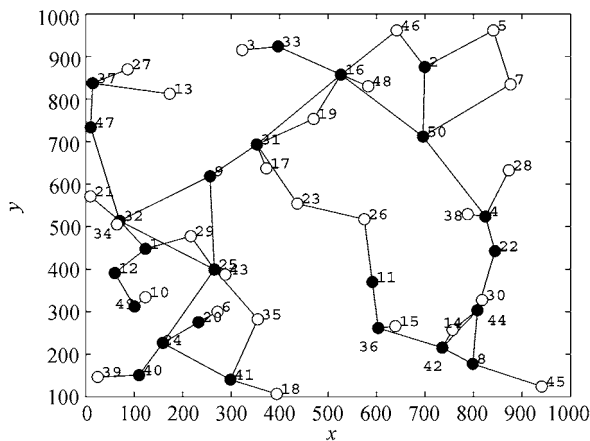
Fig.5. A Nash Equilibrium point in topology control game.

Fig.6 shows another NE point of the TCFORCE algorithm for the same initial topology. Therefore by changing the order in which nodes take turn in updating their strategies the induced topology and the power level and relay role of the nodes have changed. For example nodes 7 and 28 on the right side of the topology in the NE of Fig.6 are relay nodes and in the NE of the Fig.5 are not. This fact shows the roles of nodes' turn in their relay role and power level. Fig.7 shows that in different NE points of the TCFORCE algorithm sum of the power levels of the nodes are different.
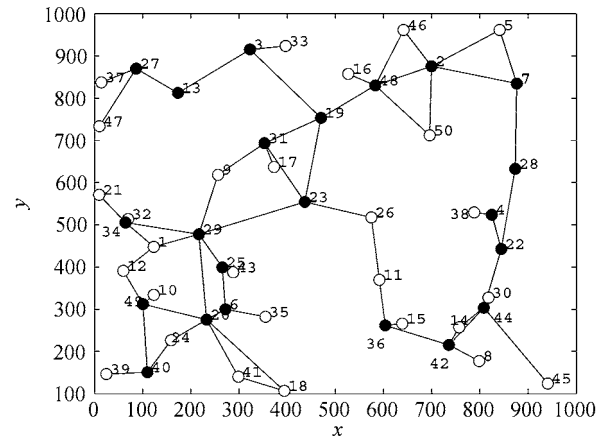

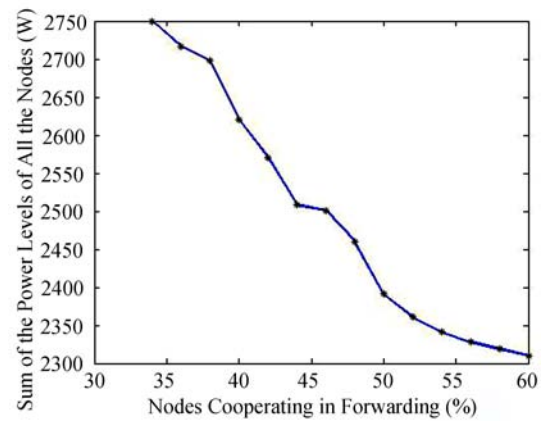Fig.6. A Nash Equilibrium point in topology control game.

Fig.7. Energy-efficiency (sum of power level of nodes) for different NE with different number of relay nodes.

This experiment is based on 20 different NE points. An interesting point in Fig.7 is that in topologies that there are more relay nodes the induced topology is more energy efficient than others. This means that if we have some cooperative nodes in the network (all the nodes are not selfish) the TCFORCE algorithm may result in more efficient topologies. In Fig.8 another energy-efficiency metric, the average minimum path cost between every pair of nodes. For this metric we have the same results as well.

Fig.9 shows the average hop count between every pair of nodes in the topologies generated by TCFORCE algorithm compared with the MIA algorithm[13] and the model presented in [12] for topologies with different numbers of nodes.
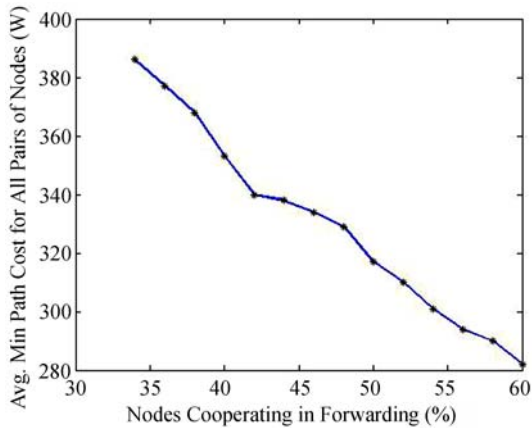
Fig.8. Energy-efficiency (average min path cost) of different NE with different number of relay nodes.
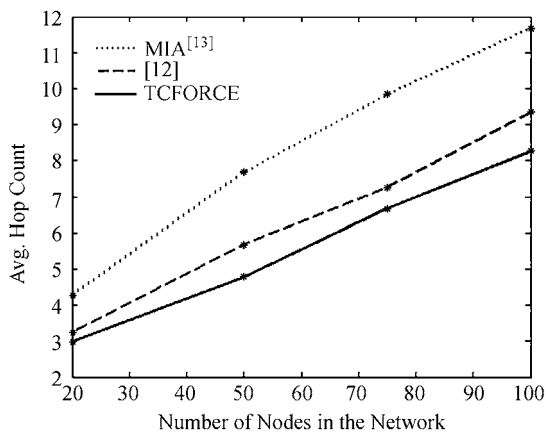

Fig.9. Average number of hops between every pair of nodes in the network for TCFORCE, [13], and [12] models.

MIA does not consider the cooperation of nodes in forwarding task and their relay role. Thus, in MIA nodes aggressively choose their lowest power level to connect to network (nodes try to choose their nearest neighbors). In the model presented in [12], nodes consider the level of cooperation of other nodes in the forwarding task in choosing their power level to connect to the topology.

In [12] in some cases nodes choose higher power levels to alleviate the high cost of transmitting their packets through their selfish neighbors. In TCFORCE, non-cooperative behavior of nodes is stricter (they are assumed to be selfish in both topology control and packet forwarding) and thus nodes have to choose even higher power levels to be connected to other nodes in the network.

In the results presented in Fig.10 the packet delivery ratio (throughput) of the network is compared between three mentioned models. We used ns-2 simulator to simulate 25 traffic flows between different pairs of

nodes in the network. We also used DSR as our routing protocol over the network. We did these experiments over the networks with 25, 50, 75, and 100 nodes. We assume selfish nodes in TCFORCE model do not forward any packet for other nodes but for the model presented in [12] and MIA[13] we consider 50% of nodes (randomly selected) are selfish and they will forward 60% of packets for others. Therefore the selfishness in TCFORCE is even more severe than in the case of [12]. Fig.10 shows that TCFORCE has better packet delivery ratio since it considers the packet forwarding behavior of nodes in topology control design and it mainly constructs topologies with backbones consisting of relay nodes.

In order to have a comparison on the energy consumption of these three models we evaluate the total energy consumed by all the nodes in the network for transmitting packets in 500 seconds with the same scenario as before (see Fig.11). We assume that dropped packets will be retransmitted and we consider the
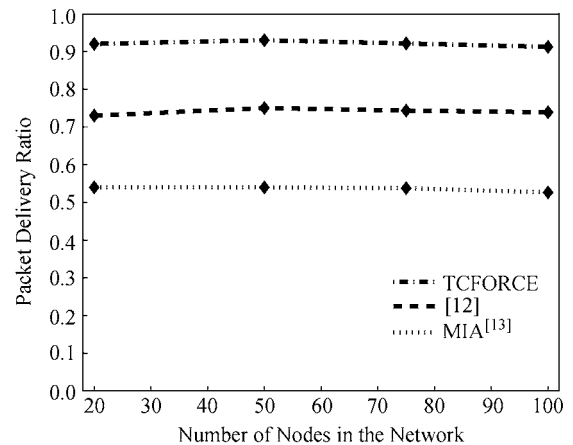

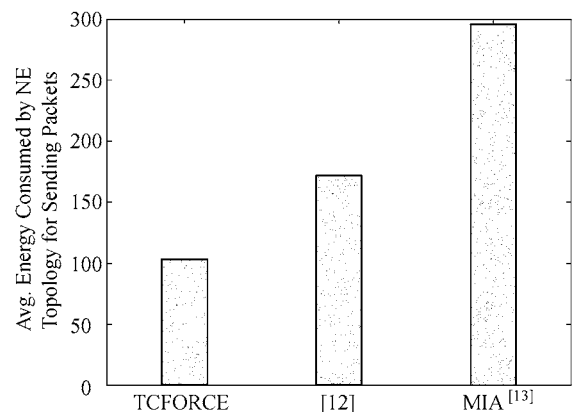Fig.10. Packet delivery ration (throughput) for TCFORCE, [13], and [12] models.

Fig.11. Average energy consumed by the resulted topologies of TCFORCE, [13], and [12] models.

retransmission cost in the energy consumption of the network. The results are the average of 50 run over the same topology.

Therefore TCFORCE model has totally reduced the energy consumption in transmitting packets by having paths of cooperative nodes in the topology. In this way TCFORCE has more reliable paths and higher packet delivery ratio.

8 Conclusions and Future Work

In this paper we have studied the cooperation problem in topology control and packet forwarding jointly in a single problem. We extended the topology control framework in order to consider the relay role of the nodes in constructing network topologies. We proved that our game model is an ordinal potential game. We also showed that the presented algorithm based on the game model, TCFORCE algorithm, preserves network connectivity and the topologies induced by this algorithm are locally energy efficient. In the proposed game model we used the preexisting concept in the problem (priority) to give nodes the incentive to cooperate rather than using credit or other micropayments, which is an advantage of our model. Our joint problem although caused more complexities but resulted in more reliable and energy efficient topologies. The drawback of this model and also other presented models in this field is relying on complete information that forces large overhead on the network. By modeling the game as an incomplete information game, we will be closer to applicability of the models in real systems.

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